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THE BOY'S ARITHMETIC.

PART I.

BY THE REV.

CHARLES ARNOLD, M.A.

RECTOR OF TINWELL,

AND LATE FELLOW OF CAIUS COLLEGE, CAMBRIDGE.

Second Edition.

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ST. JOHN'S SQUARE.

THE object of this book is twofold: (1) To make Arithmetic more easy to little boys, by enabling them to understand it: (2) To prevent their forgetting a Rule as soon as they have entered upon a new one,—a circumstance which every Teacher has often had to lament.

Each Rule is therefore explained in a familiar manner; and of the examples in each Exercise some belong to the Rule which has just been explained, and others to the Rules which have gone before.

IN this second Edition of the Boy's Arithmetic some slight improvements have been made; and some errors noticed in the previous Edition have been corrected. A Chapter has been added on Tare and Tret, to meet the wishes of some masters of National and Commercial Schools from whom the Author has received testimonies as to the value of the work.

C. A.

TINWELL,
Aug. 12, 1850.

NUMERAL LETTERS.

I	1
II	2
III	3
IV	4
V	5
VI	6
VII	7
VIII	8
IX	9
X	10
XI	11
XII	12
XIII	13
XIV	14
XV	15
XVI	16
XVII	17
XVIII	18
XIX	19
XX	20
XXI	21
XXII	22
XXIII	23
XXIV	24
XXV	25
XXX	30
XL	40
L	50
LX	60
LXX	70
LXXX	80
XC	90
C	100
CX	110
CXX	120
CXXX	130
CXL	140
CL	150
CLX	160
CLXX	170
CLXXX	180
CXC	190
CC	200
CCC	300
CCCC	400
D	500
DC	600
DCC	700
DCCC	800
DCCCC	900
M	1000
MC	1100
MCC	1200
MCCC	1300
MCCCC	1400
MD	1500
MDL	1550

ARITHMETIC FOR BOYS.

Arithmetic is the art of calculating numbers: it includes

Notation, or the art of expressing any quantity by figures; Numeration, or the art of reading numbers which have been regularly noted; and a variety of rules by which calculations are to be made.

Its principal rules are Addition, Subtraction, Multiplication, and Division, on which all the higher calculations depend.

ON NOTATION AND NUMERATION.

Certain signs have been used to express *numbers* from one to nine. Thus, 1 we call one: 2 we call two: 3, three: 4, four: 5, five: 6, six: 7, seven: 8, eight: 9, nine: and 0 we call cipher, or nought.

By these figures, differently placed, we can express any number we wish.

To express a number greater than nine, we must, however, give a different value to each figure, according to the place in which it stands; and in counting its place we begin always at the right hand; and when we find a figure in the second place, we give it ten times the value which it had in the first place: that is, we call it so many *tens*, instead of so many *ones*.

Thus, 2 signifies only *two;* that is, two *ones:* but the 2 in 28, or 23, or 20, stands for *two* "*tens*," that is *twenty*.

If the 2 were in the *third* place, counting from the right hand, it would mean *two* "*hundreds*," and so on.

The following table will show the value of a figure in each place from the first to the ninth :—

Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
9	8	7	6	5	4	3	2	1

A figure in the first, or *units' place*, has only its simple value: that is, it means so many *ones.* In the second, or *tens' place*, it has ten times its simple value : that is, it means so many *tens.* In the third, or *hundreds' place*, it has a hundred times its simple value : that is, it means so many *hundreds.* In the fourth, or *thousands'* place, it has a thousand times its simple value: that is, it means so many *thousands :* and so on.

Thus in the number 1844, the 4 in the first place, beginning at the right hand, has only its simple value, and signifies four *ones ;* that is, *four ;* the 4 in the *second*, or *tens' place*, has *ten* times its simple value, and signifies four *tens*, that is *forty :* the 8 in the *third*, or *hundreds' place*, has a *hundred* times its simple value, and signifies eight *hundreds ;* that is, *eight hundred :* and the 1 in the *fourth*, or *thousands'* place, has a *thousand* times its simple value, and signifies *one thousand ;* and the whole number, reading from left to right, is one *thousand*, eight *hundreds*, four *tens*, and four *ones :* that is, ONE THOUSAND EIGHT HUNDRED AND FORTY-FOUR.

OBSERVE. In determining the value of any figure, its place is to be counted, beginning at the first figure on the *right-hand side ;* but in *reading* the number (when the value of each figure is found), we begin with the first figure on the *left-hand side.*

If it be required to write down the number *one thousand eight hundred and forty-four*, we must proceed thus :

To express *one thousand*, we must have the figure 1 in the fourth, or thousands' place: to express *eight hundred* we must have the figure 8 in the third, or hundreds' place: to express *forty* (which is four *tens*), we must have the figure 4 in the second, or tens' place; and to express *four*, we must have the figure 4 in the *first* place, that of *units* or *ones:* and the number so noted down will be 1844.

Suppose we had to write down the following numbers:—

1. Sixteen.

This number being greater than nine, we shall require a figure in the place of tens, or second place. Now one *ten*, with *six* added to it, will make *sixteen;* we must therefore have 1 in the place of *tens*, and 6 in the place of units; and the number properly noted will be 16.

2. Ten.

Here we must have 1 in the place of tens; and (as this is the whole number to be expressed) we require *nothing* in the place of units, and therefore we put in that place a cipher, or *nought:* and the number *ten* expressed in figures is 10.

The cipher is required to bring the 1 into the place of tens: that is to say, to make its place be the *second* place.

3. Thirty-six.

Thirty-six is made up of three *tens* and six *ones.*

Here we want *three tens* to make *thirty*, and must therefore put 3 in the place of tens; and 6 in the place of *units* will represent six; and the number properly noted will be 36.

Exercise 1.

Write in figures the following numbers[1]:—

1. Twenty-nine. 2. Thirty-four. 3. Forty-three.
4. Sixty-seven. 5. Eighty-four. 6. Thirteen.
7. Seventy. 8. Eighty-one. 9. Seventy-six.
10. Fifty-four. 11. Sixty-eight. 12. Forty-three.
13. One hundred and twenty-two.
14. Seven hundred and forty-eight.
15. Five hundred and sixteen.

---

[1] Divide the number that is greater than 9 into as many *tens* as it consists of.

Write in words the following numbers :—

12, 13. 18. 17. 40. 81. 58. 39. 75. 27. 37. 53. 120. 213.

[Look at the table on page 2.]

Obs. The *fourth*, *fifth*, and *sixth* places are all places of so many *thousands*.

A figure in the *fourth* place represents so many *one* thousands; that is, so many thousands.

A figure in the *fifth* place represents so many *tens* of thousands; that is, so many *ten thousands*.

A figure in the *sixth* place represents so many *hundreds* of thousands; that is, so many *hundred thousands*.

Hence, places 1, 2, 3, are respectively so many *units* (or *ones*), *tens*, and *hundreds*. 4, 5, 6, are also respectively so many *units* (or *ones*), *tens*, and *hundreds;* but so many *units*, *tens*, and *hundreds of* THOUSANDS.

Suppose we have to express in figures the following number :—

One hundred and twenty-three thousand four hundred and fifty-six.

Here we have first to note down *one hundred thousand;* which is done by putting 1 in the *sixth* place, or place of hundreds of thousands: we then have to note down *twenty thousand;* which is done by putting 2 in the *fifth* place, or place of tens of thousands: we next have to note down *three thousand;* which is done by putting down 3 in the *fourth* place, or place of thousands. *Four hundred* will be represented by 4 in the *third* place, or place of hundreds; *fifty* by 5 in the *second* place, or place of tens; and *six* by 6 in the *first*, or units' place; and the number properly noted down will be 123,456.

Exercise 2.

Write in figures the following numbers :—

1. One thousand one hundred and seventy-four.
2. Eighteen thousand one hundred and forty-five.
3. Twenty-seven thousand nine hundred and twelve.

4. Eight hundred and fourteen thousand two hundred and sixty-seven.

5. Nine thousand one hundred and forty-nine.

6. One hundred and forty-seven thousand one hundred and forty-two.

Write in words the following numbers :—

119. 217. 814. 6312. 1814. 715. 826.

[Look at table on page 2.]

Observe that places 7, 8, 9, are *units*, *tens*, and *hundreds ;* but units of *millions ;* tens of *millions ;* hundreds of *millions.*

> " The *fourth* place is the first to *thousands* given :
> The class of *millions* must begin with *seven.*"

Let it be required to express in figures the following number :—

Two hundred and twenty-one millions one hundred and sixty thousand four hundred and thirty-seven.

We begin here with hundreds of millions, and shall therefore have 9 places of figures.

Two hundred millions will be represented by 2 in the *ninth* place, or place of hundreds of millions; *twenty millions* by 2 in the *eighth* place, or place of tens of millions ; *one million* by 1 in the *seventh* place, or place of millions. *One hundred thousand* will be represented by 1 in the sixth place, or place of hundreds of thousands : *sixty thousand* by 6 in the *fifth* place, or place of tens of thousands ; *three thousand* by 3 in the *fourth* place, or place of thousands. *Four hundred* will be represented by 4 in the *third* place, or place of hundreds ; *thirty* by 3 in the *second* place, or place of tens; and *seven* by 7 in the *first* place, or place of units ; and the number properly noted down will be 221,163,437.

Exercise 3.

Write in figures the following numbers :—

1. Two millions eight hundred and forty-one thousand four hundred and fourteen.

2. Eight millions nine hundred and sixty-seven thousand two hundred and thirteen.

3. Five millions seven hundred and twenty-five thousand one hundred and seventy-nine.

4. Sixty-eight millions two hundred and forty-two thousand six hundred and seventy-four.

5. One hundred and eighty-seven millions nine hundred and twenty-four thousand five hundred and sixty-one.

6. Three hundred and forty-six millions five hundred and ninety-seven thousand eight hundred and sixty-three.

7. Write in words the following numbers :—

394. 6162. 36814. 2714. 84216. 5798. 624148.

But sometimes we meet with numbers very much larger than any of those which have been introduced into these exercises; and to enable us to read, and to write them, we must carry up the table, beginning with units, much higher.

Quadrillions.						Trillions.						Billions.						Millions.						Units.					
Hundreds of thousands of	Tens of thousands of	Thousands of	Hundreds of	Tens of	(Units of quadrillions, that is) quadrillions	Hundreds of thousands of	Tens of thousands of	Thousands of	Hundreds of	Tens of	(Units of trillions, that is) trillions	Hundreds of thousands of	Tens of thousands of	Thousands of	Hundreds of	Tens of	(Units of billions, that is) billions	Hundreds of thousands of	Tens of thousands of	Thousands of	Hundreds of	Tens of	(Units of millions, that is) millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Units
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1

In this table the places of figures are divided into classes of six. The first class carries us up from units to hundreds of thousands, and may be called the *units'* class.

The second class begins with millions, and carries us up to hundreds of thousands of millions, and is called the *millions' class*. The third class carries us up from billions to hundreds of thousands of billions, and is the billions' class, and so on for the fourth, or trillions' class; the fifth, or quadrillions' class, &c. &c. And in the same way the table may be carried to any height, and, consequently, any number may be expressed, or *noted*.

In each of these classes the terms and order are exactly the same, except that the *name of the class* is always introduced in the first place of each.

Thus, in the first class, the 1 is *one* UNIT; in the second class, the 1 is *one* MILLION; in the fifth class, it is *one* QUADRILLION, &c.

So in the first class, the 2 is *two* TENS, or TWENTY: in the second class it is *two* TENS OF MILLIONS, or TWENTY MILLIONS: and in the fourth class, it is two TENS OF TRILLIONS, or *twenty* TRILLIONS, &c.

In the first class, the 4 is *four thousands of* UNITS, or FOUR THOUSAND; in the second class, it is *four thousands of* MILLIONS, or FOUR THOUSAND MILLIONS, &c.

In the first class, the 5 *is five tens of thousands of* UNITS, or FIFTY THOUSAND, because five tens = fifty. In the second class, the 5 is *five tens of thousands of* MILLIONS, or *fifty thousand* MILLIONS. And in the fourth class it is *five tens of thousands of* TRILLIONS, or *fifty thousand* TRILLIONS.

The first class might be divided into two: the first three places being units, and the next three places being thousands, as it is at once seen that in the fourth, fifth, and sixth places we only substitute the word *thousands* for the word *units* in the first, second, and third places. But the division into classes of six places in each will make the table more regular, and the mode of calculation clearer to the student.

Exercise 4.

1. What is the value of the 4 in the 1st class?
2. What is the value of the 3 in the second class?
3. What is the value of the 5 in the fourth class?

Write down in figures the following numbers:—

4. Thirty-four. 5. Forty-nine.
6. Sixty-three. 7. Seventy-five.
8. One hundred and twenty-three thousand two hundred and sixteen.
9. Eight thousand four hundred and twenty-one.

Write in words the following numbers:—

29. 234. 8142. 91763.

Exercise 5.

1. What is the value of 2 in the third class?
2. What is the value of 6 in the fifth class?
3. What is the value of 4 in the third class?

Write in figures the following numbers:—

4. Twenty-four thousand five hundred and sixty-three.
5. One hundred and eighty-seven thousand four hundred and twenty-eight.
6. Write in words the following numbers:—

7141271. 832369. 714686127.

In order to enable us to to read off any number more easily, it is usual to place a mark after every three figures, counting from the place of units, thus,

21,479,847,216

by which means we easily see to which class any figure belongs, and what place it occupies in that class; and can therefore at once determine its value.

In the above number there are only eleven figures; consequently the highest figure is in the fifth place of the second, or millions' class, and corresponds with tens of thousands of millions; and being a 2, it represents two tens of thousands of millions, or *twenty thousand millions.* The 1 is in the fourth place of the second, or millions' class, and represents *one thousand millions.* The 4, in the third place of the second or millions' class, represents *four hundred millions:* the 7, in the second place of the same class, represents 7 tens of millions, or *seventy millions:* the 9, in the first place of the same class, represents *nine millions:* the 8, in the sixth place of the first class, represents *eight hundred thousand:* the 4, next to it, four tens of thousands, or *forty thousand:* the 7, *seven thousand:* the 2, *two* *hundreds:* the 1, one ten, or *ten:* and the 6 has only its *simple value.* The whole number, therefore, in writing will be

"Twenty-one thousand four hundred and seventy-nine millions, eight hundred and forty-seven thousand, two hundred and sixteen."

Exercise 6.

Express in writing the following numbers:—

27141456. 8642121. 796143. 8274.
694142. 684871464.

1. What is the value of 8, when standing in the fourth place of the second class in the Numeration Table?
2. What would be the value of 5 in the sixth place of the first class?
3. What would be the value of 7 in the third place of the second class?
4. What would be the value of 6 in the fifth place of the second class?
5. What would be the value of 9 in the fourth place of the third class?

In expressing any number which does not descend to the *place of units,* care must be taken to supply the number of ciphers which is necessary to bring the significant figures into their proper places, in the class to which each belongs.

Thus, to express two hundred and sixty-five millions, we must set down the same figures as if we had merely to express two hundred and sixty-five, viz., 265: and then we bring them into the millions' class, by adding six ciphers, to bring the 5 to the seventh place, thus, 265,000,000.

To express 265 billions, we must use the same three significant figures, followed by twelve ciphers, to bring them into the billions' class, thus, 265,000,000,000,000.

Exercise 7.

Write down in figures,

1. Two hundred and thirty-one thousand.
2. Three hundred and forty thousand.
3. One million four hundred thousand.

4. Thirty-three millions.
5. Five hundred and forty millions.
6. Four thousand three hundred and ten millions.

Express in writing,

270. 2800. 141000. 810000. 970000. 60.

Exercise 8.

Write down in figures,

1. Four billions.
2. Fifty-seven billions.
3. Eight hundred and twenty-nine billions.
4. Nine thousand four hundred billions.
5. Twenty-three trillions.
6. Three hundred and thirty-three trillions.

Write in words,

840000000. 2917600000. 864000000.

The same care must be taken to put a cipher in every place in each class in which there is no significant figure.

Thus, to write down two thousand and fourteen millions, we must have a cipher between the 2 in the place of thousands, and the 1 in the place of tens; thus, 2014, &c., because there is no significant figure required in the place of hundreds in that class; and if we omitted to introduce a cipher, the 2 would come next to the 1, (thus, 214, &c.) and would represent *two hundred* instead of *two thousand*: but by setting down a cipher in the place of hundreds (2014, &c.), the 2 is made to occupy its proper place in its class. And, having thus determined the significant figures and their places, we bring the whole into the millions' class by adding six ciphers to it, thus, 2,014,000,000.

To express two thousand and fourteen millions twelve thousand and seventy, we shall require ciphers in four different places, viz., in the *hundreds of millions*, or 9th place; in the *hundreds of thousands*, or 6th place: in the *hundreds'* place; and in the *units'* place. And when the proper significant figures are introduced into their respective places, the sum will be thus noted, 2,014,012,070.

Exercise 9.

Write down in figures,

1. Eighty-seven thousand and forty.
2. One hundred and ninety thousand and twenty-four.
3. Eight hundred thousand one hundred and two.
4. Four millions and seventy-six.

Express in writing,

80101. 907024. 60027. 6801070. 9541701.

Exercise 10.

Write in figures,

1. Eight millions one thousand and four.
2. One hundred and twenty millions thirty-four thousand.
3. Eight hundred and four millions eleven thousand and sixty-three.
4. Nine hundred thousand millions nine hundred thousand and forty-two.

Express in writing,

80008. 9002001. 70401107. 8604012003.

CHARACTERS.

= Equal.
— Minus, or less.
+ Plus, or more.
× Multiplied by.
÷ Divided by.

¼ One quarter.
⅓ One Third.
½ One Half.
¾ Three Quarters.

SIMPLE ADDITION

Is the method of collecting into *one sum* any numbers.

If the numbers are all under 10, or *less* than 10, place them one under another, add the two first together; to their sum add the third figure; to the sum thus found add the fourth figure, and so on to the top of the column; place the units' figure of the amount under the column, and the tens' figure to its left side.

Thus, let it be required to add together, 2. 4. 5. 7. 9. 8. 4. and 3.

$$\begin{array}{r}2\\4\\5\\7\\9\\8\\4\\3\\\hline 42\\\hline\hline\end{array}$$

The figures must be placed one under another, and a line drawn below them. The figures are then to be added thus, beginning with the lowest. 3 and 4 make 7: 7 and 8 make 15: 15 and 9 make 24: 24 and 7 make 31: 31 and 5 make 36: 36 and 4 make 40: 40 and 2 make 42. And the result must be written down below, the two, which belongs to the place of units, being put under the column of figures, and the 4, which must be in the *place of tens* to represent 40, being one place further to the left.

ADDITION TABLE.

1 and 1 are 2	2 and 1 are 3	3 and 1 are 4
1 .. 2 .. 3	2 .. 2 .. 4	3 .. 2 .. 5
1 .. 3 .. 4	2 .. 3 .. 5	3 .. 3 .. 6
1 .. 4 .. 5	2 .. 4 .. 6	3 .. 4 .. 7
1 .. 5 .. 6	2 .. 5 .. 7	3 .. 5 .. 8
1 .. 6 .. 7	2 .. 6 .. 8	3 .. 6 .. 9
1 .. 7 .. 8	2 .. 7 .. 9	3 .. 7 .. 10
1 .. 8 .. 9	2 .. 8 .. 10	3 .. 8 .. 11
1 .. 9 .. 10	2 .. 9 .. 11	3 .. 9 .. 12
1 .. 10 .. 11	2 .. 10 .. 12	3 .. 10 .. 13
1 .. 11 .. 12	2 .. 11 .. 13	3 .. 11 .. 14
1 .. 12 .. 13	2 .. 12 .. 14	3 .. 12 .. 15

4 and 1 are 5	5 and 1 are 6	6 and 1 are 7
4 .. 2 .. 6	5 .. 2 .. 7	6 .. 2 .. 8
4 .. 3 .. 7	5 .. 3 .. 8	6 .. 3 .. 9
4 .. 4 .. 8	5 .. 4 .. 9	6 .. 4 .. 10
4 .. 5 .. 9	5 .. 5 .. 10	6 .. 5 .. 11
4 .. 6 .. 10	5 .. 6 .. 11	6 .. 6 .. 12
4 .. 7 .. 11	5 .. 7 .. 12	6 .. 7 .. 13
4 .. 8 .. 12	5 .. 8 .. 13	6 .. 8 .. 14
4 .. 9 .. 13	5 .. 9 .. 14	6 .. 9 .. 15
4 .. 10 .. 14	5 .. 10 .. 15	6 .. 10 .. 16
4 .. 11 .. 15	5 .. 11 .. 16	6 .. 11 .. 17
4 .. 12 .. 16	5 .. 12 .. 17	6 .. 12 .. 18

7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
7 .. 2 .. 9	8 .. 2 .. 10	9 .. 2 .. 11
7 .. 3 .. 10	8 .. 3 .. 11	9 .. 3 .. 12
7 .. 4 .. 11	8 .. 4 .. 12	9 .. 4 .. 13
7 .. 5 .. 12	8 .. 5 .. 13	9 .. 5 .. 14
7 .. 6 .. 13	8 .. 6 .. 14	9 .. 6 .. 15
7 .. 7 .. 14	8 .. 7 .. 15	9 .. 7 .. 16
7 .. 8 .. 15	8 .. 8 .. 16	9 .. 8 .. 17
7 .. 9 .. 16	8 .. 9 .. 17	9 .. 9 .. 18
7 .. 10 .. 17	8 .. 10 .. 18	9 .. 10 .. 19
7 .. 11 .. 18	8 .. 11 .. 19	9 .. 11 .. 20
7 .. 12 .. 19	8 .. 12 .. 20	9 .. 12 .. 21

10 .. 1 .. 11	11 and 1 are 12	12 and 1 are 13
10 .. 2 .. 12	11 .. 2 .. 13	12 .. 2 .. 14
10 .. 3 .. 13	11 .. 3 .. 14	12 .. 3 .. 15
10 .. 4 .. 14	11 .. 4 .. 15	12 .. 4 .. 16
10 .. 5 .. 15	11 .. 5 .. 16	12 .. 5 .. 17
10 .. 6 .. 16	11 .. 6 .. 17	12 .. 6 .. 18
10 .. 7 .. 17	11 .. 7 .. 18	12 .. 7 .. 19
10 .. 8 .. 18	11 .. 8 .. 19	12 .. 8 .. 20
10 .. 9 .. 19	11 .. 9 .. 20	12 .. 9 .. 21
10 .. 10 .. 20	11 .. 10 .. 21	12 .. 10 .. 22
10 .. 11 .. 21	11 .. 11 .. 22	12 .. 11 .. 23
10 .. 12 .. 22	11 .. 12 .. 23	12 .. 12 .. 24

Exercise 11.

1. Add together 8. 7. 4. 1. 5. 7. 9. 6. 4.
2. Add together 3. 4. 9. 8. 2. 6. 5. 9. 7.
3. Add together 6. 2. 7. 1. 4. 7. 6. 1. 3.

Write down in figures,

4. One hundred and seventy-five thousand.
5. One million eight hundred thousand and two.

Express in writing,

50416. 82479065. 2000304. 6001010200.

The sign +, called plus (more), placed between two figures or numbers, means that they are to be added together.

The sum of any numbers added together is sometimes called the *amount.*

Exercise 12.

1. What is the sum, or amount, of $4 + 6 + 3$?
2. What is the amount of $8 + 7 + 3 + 2 + 4 + 7 + 9$?
3. What is the sum of $3 + 2 + 4 + 9 + 8 + 1$?
4. Add together 4, 2, 1, 7, 9, 1, 5.
5. Add together 6, 8, 4, 2, 1, 7, 9, 5, 4.

Write in figures,

6. Sixteen thousand and seventy-four.
7. Two millions and twenty-four thousand.
8. Eighty-four millions twenty-nine thousand and fourteen.

When the numbers to be added contain more than one figure each, they must be set down one under another, in such a manner that the units fall under units, the tens under tens, &c.

Thus, if it be required to add together 24, 13, 46, and 9. We write down the numbers so that the 4, 3, 6, and 9 fall under each other in the units' column, and the 2, 1, and 4, under each other in the place of tens.

$$\begin{array}{r} 24 \\ 13 \\ 46 \\ 9 \\ \hline 92 \\ \hline \end{array}$$

The first column is then added as in the last exercises; 9 and 6 make 15: 15 and 3 make 18: 18 and 4 make 22 : the figure therefore to be put down in the units' place is 2: and to represent the *twenty* a 2 is required in the place of tens : but it must not be set down yet, because there are three other figures in the place of tens to be added together: the 2, therefore, belonging to the place of tens, must be added *with* these : thus, 2 and 4 make 6 : 6 and 1 make 7: 7 and 2 make 9.

The 2 is said to be *carried* from the column of units to that of *tens*.

Observe, if the amount is above 9 and under 20, 1 is carried: from 20 to 29 (inclusive) 2 are carried: from 30 to 39, 3 are carried, &c. The reason is this : 1, 2, and 3, in the place of tens, really express ten, twenty, and *thirty*.

Exercise 13.

1. Add together 15, 17, 25, 31, 9.
2. Add together 16, 15, 9, 18, 21.
3. Add together 12, 20, 17, 6.
4. What is the amount of $8 + 9 + 16 + 24 + 32$?
5. What is the amount of $12+34+54+32+11+23+14+21$?
6. What is the sum of 46, 56, 43, 94, 76, 89, 43, 21?
7. What is the value of 6 in the third place of the fourth class?
8. What is the value of 7 in the fourth place of the third class?
9. What is the value of 9 in the second place of the third class?

When the figures in the second column amount to more than 9, the same process must be pursued of carrying 1 for every *ten* to the place of hundreds.

And generally: every column is added in the same way, and, in doing the work, is treated precisely as if it were a column of units; the *value* of a figure always depending on the PLACE which it occupies. And as every figure, when removed one place further to the left, has *ten times* the value which it had in the preceding place, in every column we must carry 1 for ten, 2 for twenty, 3 for thirty, &c. &c.

Exercise 14.

1. Why do you carry only 3 from the units' column to the tens' column, when the amount of the units' column is 36?
2. Why do you carry 4 from the tens to the hundreds' column, when the amount of the tens' column is 47?
3. Add together 27, 49, 86, and 33.
4. Add together 31, 54, 72, and 89.
5. What is the amount of $6+41+97+12+7+11$?
6. What is the amount of $9+8+17+29+3+97$?
7. What is the amount of $12+29+34+41+9+84+73$?
8. A man aged 76, and his wife aged 74, had 5 children, viz.: John 50, William 48, Mary 40, Jane 39, and Henry 34; what were the united ages of the family?

Exercise 15.

1. Add together 18, 14, 91, 23, 36, 40.
2. Add together 13, 19, 10, 24, 36, 42.
3. What is the sum of 459, 145, 712, 523, 604, 589?
4. Add together 925, 516, 154, 239, 915, 426, 543, 754.
5. A farmer has seven fields, one containing 25 acres, another 57, another 18, another 25, another 15, another 43, and another 9 acres. How many acres has he?
6. What would you carry to the tens if the units amounted to 67? and what to the hundreds, if the column of tens amounted to 89?

Sometimes the amount of a column is more than one hundred, as 108; in that case the 8 must be put in the units' place, and 10 must be carried, because 10 in the column of tens is equal to one hundred: if it be above 109, as 114, 11 must be carried, for 11 in the place of tens is equal to *one hundred and ten.* So also for 125 we should carry 12, &c.

Whatever, therefore, be the amount of a column, set down the right-hand figure of that amount (properly noted) under the column, and add the number which is represented by the remaining figures to the next column on the left.

Exercise 16.

1. Add together 887, 981, 792, 485, 84, 96, 81.
2. Add together 198, 96, 182, 79, 87, 94, 60, 96, 79, 88, 199, 797.
3. Add together 84, 102, 3045, 847, 9815, 372941, 8271, 39.
4. Add together 176543, 6789195, 2949, 87, 6527, 68491, 2479812.
5. Express in writing the answers of the four preceding sums.

Exercise 17.

1. What is the sum of 87, 9876, 45372, 91, 89427, 9717195?
2. What is the sum of 21414, 8987471, 382, 984, 10, 6, 81414275?
3. Three men enter into partnership: the first has fifty

thousand pounds, the second forty-nine thousand eight hundred and four pounds, and the third has seventy thousand one hundred and twenty-six pounds; what is their joint capital?

4. If A has 56*l.*, B 104*l.*, C 274*l.*, D 1390*l.*, E 7003*l.*, F 1500*l.*, G 998*l.*, and H 1000*l.*, how much is the joint amount of their money?

5. When will the lease of a field expire which was granted in the year 1844, for a term of 48 years?

Two or three methods of proving whether the work is rightly performed are generally taught.

One is this. When all the numbers have been properly set down under each other, and have been added together, and the amount has been placed below, then draw a line under the upper line of figures, and add together all the lines, omitting this upper one; set the amount under the former one; then add together this second amount and the upper line, and if the work be right this third amount will correspond with the first.

81415
. . . .
21714
36512
49798

189439 first amount.

108024 amount of the 3 lower lines.

189439 second amount and upper line together.

This last amount, being the sum of the three lower lines added to the upper line, is the amount of the four lines together; and, as it corresponds with the first amount, we *conclude* that the work is right.

But the easiest, and perhaps the best, way is to go over each column a second time, beginning at the top and going downwards, instead of beginning at the bottom and going upwards: and if the results correspond, there is very little doubt but the work is right.

Prove the answers in the following sums.

Exercise 18.

1. Add together 8141407, 912154, 8767, 9151428, 379761.
2. Add together 2717, 8759214, 9141512, 61, 79, 846, 27158.
3. Add together 2141, 981, 641, 3782, 541456, 71415, 8021, 98754.
4. What is the amount of sixty thousand and ten: one hundred and seven thousand and eighty: six millions three thousand and five: twenty millions one hundred and nine thousand and four?
5. What is the amount of twelve millions seven thousand and forty: twenty-nine millions five hundred thousand and twenty-three: one hundred and twenty-three millions six hundred and nine?
6. Add 9876, 5943, 6876, 3459, 8634, 7698, 8749, 2176.
7. Add 4716, 5964, 3186, 5418, 9654, 3219, 8654, 4789.
8. Add 467543, 6487, 7429, 73862, 65487, 5321, 9321, 7412.

SUBTRACTION

Is the method of finding the *difference* between two numbers.

Place the smaller number below the greater, in such a manner that the units are under units, tens under tens, &c.; begin at the right hand; subtract each figure in its turn from the one above it, and place the remainder, or difference, below it.

Thus,	From 21415	From 812976
	Take 10213	Take 610234
	11202	202742

The sign —, minus (less), between two numbers, means that the latter is to be subtracted from the former.

SUBTRACTION TABLE.

1 from 2 leaves 1
1 .. 3 .. 2
1 .. 4 .. 3
1 .. 5 .. 4
1 .. 6 .. 5
1 .. 7 .. 6
1 .. 8 .. 7
1 .. 9 .. 8
1 .. 10 .. 9
1 .. 11 .. 10
1 .. 12 .. 11
1 .. 13 .. 12

2 from 3 leaves 1
2 .. 4 .. 2
2 .. 5 .. 3
2 .. 6 .. 4
2 .. 7 .. 5
2 .. 8 .. 6
2 .. 9 .. 7
2 .. 10 .. 8
2 .. 11 .. 9
2 .. 12 .. 10
2 .. 13 .. 11
2 .. 14 .. 12

3 from 4 leaves 1
3 .. 5 .. 2
3 .. 6 .. 3
3 .. 7 .. 4
3 .. 8 .. 5
3 .. 9 .. 6
3 .. 10 .. 7
3 .. 11 .. 8
3 .. 12 .. 9
3 .. 13 .. 10
3 .. 14 .. 11
3 .. 15 .. 12

4 from 5 leaves 1
4 .. 6 .. 2
4 .. 7 .. 3
4 .. 8 .. 4
4 .. 9 .. 5
4 .. 10 .. 6
4 .. 11 .. 7
4 .. 12 .. 8
4 .. 13 .. 9
4 .. 14 .. 10
4 .. 15 .. 11
4 .. 16 .. 12

5 from 6 leaves 1
5 .. 7 .. 2
5 .. 8 .. 3
5 .. 9 .. 4
5 .. 10 .. 5
5 .. 11 .. 6
5 .. 12 .. 7
5 .. 13 .. 8
5 .. 14 .. 9
5 .. 15 .. 10
5 .. 16 .. 11
5 .. 17 .. 12

6 from 7 leaves 1
6 .. 8 .. 2
6 .. 9 .. 3
6 .. 10 .. 4
6 .. 11 .. 5
6 .. 12 .. 6
6 .. 13 .. 7
6 .. 14 .. 8
6 .. 15 .. 9
6 .. 16 .. 10
6 .. 17 .. 11
6 .. 18 .. 12

7 from 8 leaves 1
7 .. 9 .. 2
7 .. 10 .. 3
7 .. 11 .. 4
7 .. 12 .. 5
7 .. 13 .. 6
7 .. 14 .. 7
7 .. 15 .. 8
7 .. 16 .. 9
7 .. 17 .. 10
7 .. 18 .. 11
7 .. 19 .. 12

8 from 9 leaves 1
8 .. 10 .. 2
8 .. 11 .. 3
8 .. 12 .. 4
8 .. 13 .. 5
8 .. 14 .. 6
8 .. 15 .. 7
8 .. 16 .. 8
8 .. 17 .. 9
8 .. 18 .. 10
8 .. 19 .. 11
8 .. 20 .. 12

9 from 10 leaves 1
9 .. 11 .. 2
9 .. 12 .. 3
9 .. 13 .. 4
9 .. 14 .. 5
9 .. 15 .. 6
9 .. 16 .. 7
9 .. 17 .. 8
9 .. 18 .. 9
9 .. 19 .. 10
9 .. 20 .. 11
9 .. 21 .. 12

10 from 11 leaves 1	11 from 12 leaves 1	12 from 13 leaves 1
10 .. 12 .. 2	11 .. 13 .. 2	12 .. 14 .. 2
10 .. 13 .. 3	11 .. 14 .. 3	12 .. 15 .. 3
10 .. 14 .. 4	11 .. 15 .. 4	12 .. 16 .. 4
10 .. 15 .. 5	11 .. 16 .. 5	12 .. 17 .. 5
10 .. 16 .. 6	11 .. 17 .. 6	12 .. 18 .. 6
10 .. 17 .. 7	11 .. 18 .. 7	12 .. 19 .. 7
10 .. 18 .. 8	11 .. 19 .. 8	12 .. 20 .. 8
10 .. 19 .. 9	11 .. 20 .. 9	12 .. 21 .. 9
10 .. 20 .. 10	11 .. 21 .. 10	12 .. 22 .. 10
10 .. 21 .. 11	11 .. 22 .. 11	12 .. 23 .. 11
10 .. 22 .. 12	11 .. 23 .. 12	12 .. 24 .. 12

Exercise 19.

1. From 24 take 13.
2. From 84 take 63.
3. From 97 take 16.
4. From 27698 take 12461.
5. Add together 614, 71216, 31270, 40216, 7142, 61506.
6. What is the sum of 37141, 27298, 91412, 7021, 81, 701, 68584?

Write in figures,

7. Three hundred and five millions two thousand and twenty-seven.
8. Ninety-eight millions twenty-seven thousand two hundred.

Exercise 20.

1. From 82814 take 71803.
2. From 98279 take 66154.
3. Add together 81416, 217145, 814123, 871, 9152, 63.

Express in writing,

4. 20104, 87170102, 7010, 910104207, 6000304, 8010409107012.

When any figure in the lower line is larger than the one in the upper line from which it is to be subtracted, *ten* must be added to the upper figure, and the lower one then subtracted; and when the remainder has been set down, *one* must be added to the *next figure which has to be subtracted* before it is taken from the figure above it.

$$\text{Thus,}\qquad \begin{array}{r} 814 \\ 341 \\ \hline 473 \\ \hline\hline \end{array}$$

Here we take 1 from 4, and 3 remain; we then have to take 4 from 1, which cannot be done; we therefore add 10 to the 1, and say 4 from 11, and 7 remain: we then add 1 to the 3, and say 4 from 8, and 4 remain.

One is added to the 3 before it is subtracted to *compensate for*, or *balance*, the *ten* which was added to the 1 in the previous place of the upper line, because 1 in the place of hundreds is equal to 10 in the place of tens, and therefore TEN 10's having been added, and 1 HUNDRED taken away, the answer has not been either increased or diminished.

Adding *ten* to the 1 is called *borrowing ten;* and adding *one* to the 3 is called *carrying one :* and it must be remembered that, whenever it is necessary to borrow *ten*, it is always necessary to carry *one*. And we never borrow more than 10, nor carry more than 1, because in every part of the operation 10 in any place of figures is only equal to 1 in the next higher place.

Exercise 21.

1. From 6814 take 3633.
2. From 7181 take 6027.
3. From 6717 take 4706.
4. From 87174 take 76812.
5. What is the difference between 81714 and 97812?
6. What is the difference between 67136 and 864162?
7. What is the sum of 8141214, 3914917, 6842719, 8541216?
8. What is the sum of 214, 821, 79368, 4727145, 6959, 415, 37217164?
9. A is 64 years of age; in what year was he born?
10. B was born in the year 1799, how old is he?

Sometimes it will be necessary to borrow 10, and carry 1, in several succeeding steps of the operation.

$$\begin{array}{lr} \text{Thus,} & 1000000 \\ & 999999 \\ \hline & \ldots\ldots 1 \end{array}$$

In which example the difference is only 1.

In the units' place we have to take 9 from 0, which we cannot do: we therefore borrow 10, and taking 9 from 10, we have a remainder 1. To the next place of figures we have to carry 1, and to take, not 9, but 10, from 0: as this cannot be done, we again borrow 10, and taking 10 from 10, we have no remainder; and in the same way we proceed, taking 10 from 10 at every step, until we come to the place of millions, where we have 1 in the upper line and nothing below it; but, as we have to carry 1 from the preceding place, we have no remainder. And thus in every place of figures, except the first, there is no remainder.

The following method is generally used to *prove* whether the work is rightly performed:—

After you have subtracted the lower line from the upper, and noted down the remainder, add this remainder to the lower line, and, if the work is right, the amount will be the same as the upper line.

$$\begin{array}{llrl} \text{Thus,} & \text{From} & 814 & \\ & \text{Take} & 341 & \\ \cline{3-3} & & 473 & \text{remainder.} \\ \cline{3-3} & & 814 & \text{proof.} \end{array}$$

Exercise 22.

1. From 9712314 take 6847613.
2. From 8120704 take 5712043.
3. From 651216821 take 452162443.
4. From 7643769754362 take 6437691543127.

[redacted] world was created before Christ 4004 years: called by God B.C. 1921 years. What interval was there between the creation and the call of Abraham?

6. The Israelites left Egypt B.C. 1491. What time elapsed between the creation and their deliverance? and between the call of Abraham and their deliverance?

7. Troy was destroyed B.C. 1183: how long was this after the call of Abraham?

8. Express in figures,

One thousand nine hundred and twenty-one.
Four thousand and four.
One thousand and twelve.
Eight millions and thirty-two.

9. If twenty-one thousand four hundred be deducted from half a million, what will remain?

MULTIPLICATION

Is the method of finding the amount of any one number repeated any given number of times.

It is in fact a short way of adding the same number together several times.

Thus, if we had to add 714 to itself seven times, we might do it by addition thus:

$$\begin{array}{r} 714 \\ 714 \\ 714 \\ 714 \\ 714 \\ 714 \\ 714 \\ \hline 4998 \end{array}$$

But it is usually done by multiplication, thus:

$$\begin{array}{r} 714 \\ 7 \\ \hline 4998 \end{array}$$

MULTIPLICATION TABLE.

Twice 1 are 2	3 times 1 are 3	4 times 1 are 4
Twice 2 .. 4	3 .. 2 .. 6	4 .. 2 .. 8
Twice 3 .. 6	3 .. 3 .. 9	4 .. 3 .. 12
Twice 4 .. 8	3 .. 4 .. 12	4 .. 4 .. 16
Twice 5 .. 10	3 .. 5 .. 15	4 .. 5 .. 20
Twice 6 .. 12	3 .. 6 .. 18	4 .. 6 .. 24
Twice 7 .. 14	3 .. 7 .. 21	4 .. 7 .. 28
Twice 8 .. 16	3 .. 8 .. 24	4 .. 8 .. 32
Twice 9 .. 18	3 .. 9 .. 27	4 .. 9 .. 36
Twice 10 .. 20	3 .. 10 .. 30	4 .. 10 .. 40
Twice 11 .. 22	3 .. 11 .. 33	4 .. 11 .. 44
Twice 12 .. 24	3 .. 12 .. 36	4 .. 12 .. 48

5 times 1 are 5	6 times 1 are 6	7 times 1 are 7
5 .. 2 .. 10	6 .. 2 .. 12	7 .. 2 .. 14
5 .. 3 .. 15	6 .. 3 .. 18	7 .. 3 .. 21
5 .. 4 .. 20	6 .. 4 .. 24	7 .. 4 .. 28
5 .. 5 .. 25	6 .. 5 .. 30	7 .. 5 .. 35
5 .. 6 .. 30	6 .. 6 .. 36	7 .. 6 .. 42
5 .. 7 .. 35	6 .. 7 .. 42	7 .. 7 .. 49
5 .. 8 .. 40	6 .. 8 .. 48	7 .. 8 .. 56
5 .. 9 .. 45	6 .. 9 .. 54	7 .. 9 .. 63
5 .. 10 .. 50	6 .. 10 .. 60	7 .. 10 .. 70
5 .. 11 .. 55	6 .. 11 .. 66	7 .. 11 .. 77
5 .. 12 .. 60	6 .. 12 .. 72	7 .. 12 .. 84

8 times 1 are 8	9 times 1 are 9	10 times 1 are 10
8 .. 2 .. 16	9 .. 2 .. 18	10 .. 2 .. 20
8 .. 3 .. 24	9 .. 3 .. 27	10 .. 3 .. 30
8 .. 4 .. 32	9 .. 4 .. 36	10 .. 4 .. 40
8 .. 5 .. 40	9 .. 5 .. 45	10 .. 5 .. 50
8 .. 6 .. 48	9 .. 6 .. 54	10 .. 6 .. 60
8 .. 7 .. 56	9 .. 7 .. 63	10 .. 7 .. 70
8 .. 8 .. 64	9 .. 8 .. 72	10 .. 8 .. 80
8 .. 9 .. 72	9 .. 9 .. 81	10 .. 9 .. 90
8 .. 10 .. 80	9 .. 10 .. 90	10 .. 10 .. 100
8 .. 11 .. 88	9 .. 11 .. 99	10 .. 11 .. 110
8 .. 12 .. 96	9 .. 12 .. 108	10 .. 12 .. 120

11 times 1 are 11	12 times 1 are 12	20 times 1 are 20
11 .. 2 .. 22	12 .. 2 .. 24	20 .. 2 .. 40
11 .. 3 .. 33	12 .. 3 .. 36	20 .. 3 .. 60
11 .. 4 .. 44	12 .. 4 .. 48	20 .. 4 .. 80
11 .. 5 .. 55	12 .. 5 .. 60	20 .. 5 ..100
11 .. 6 .. 66	12 .. 6 .. 72	20 .. 6 ..120
11 .. 7 .. 77	12 .. 7 .. 84	20 .. 7 ..140
11 .. 8 .. 88	12 .. 8 .. 96	20 .. 8 ..160
11 .. 9 .. 99	12 .. 9 ..108	20 .. 9 ..180
11 ..10 ..110	12 ..10 ..120	20 ..10 ..200
11 ..11 ..121	12 ..11 ..132	20 ..11 ..220
11 ..12 ..132	12 ..12 ..144	20 ..12 ..240

The number to be multiplied is called the MULTIPLICAND.

The number by which it is to be multiplied is called the MULTIPLIER.

To multiply by a single figure.

Set down the number to be multiplied, or multiplicand; and under the *units' place* in it set down the figure by which it is to be multiplied, or the multiplier. Multiply the figure in the units' place of the multiplicand by the multiplier, and, if the product is less than 10, set it down below; but if it is more than 10, and less than 20, set down the difference between it and 10, and carry 1, to be added to the product of the next figure, when it has been multiplied by the same multiplier: if it is between 20 and 30, set down the difference between it and 20 and carry 2, &c., always carrying 1 for every ten which the product exceeds.

Thus, to multiply 714 by 7.

$$\begin{array}{r} 714 \\ 7 \\ \hline 4998 \\ \hline\hline \end{array}$$

We first multiply the 4 by 7 thus: 7 times 4 equal 28; we therefore set down 8 in the units' place, and carry 2 (for the two tens or twenty), to be added to the next product: we then multiply the 1 by 7, which equals 7, and to this we add the 2 which we carried, which makes the result 9. We therefore set down 9. We have this time nothing to carry: we then multiply the 7 by 7, which gives a product 49, and as there is nothing to be carried to this, and no more figures to be multiplied, we set down the 9 under the 7, and carry the 4 one place further to the left.

The same method must be followed for any number of figures in the multiplicand.

Exercise 23.

1. Multiply 414 by 2. And 2. 316 by 3.
3. Multiply 4178 by 4. And 4. 61764 by 5.
5. Multiply 81412 by 6. And 6. 67141 by 8.
7. Multiply 91812 by 7.
8. Printing was introduced in the year 1474; how many years has this invaluable art been known to us?
9. A boy playing at marbles lost 16, then 24, then 32, and afterwards 41. After this he won 17, 49, and 19. He began with 79. How many had he at last?

Exercise 24.

1. Multiply 6796547 by 2.
2. Multiply 3964659 by 3.
3. Multiply 3756499 by 4.
4. Multiply 87665439 by 5.
5. Multiply 7546396 by 6.
6. Multiply 8465495 by 7.
7. Multiply 8765439 by 8.
8. Multiply 79654965 by 9.
9. The yearly income of a gentleman is four thousand and twenty pounds: his son has an income of one thousand nine hundred and eighty pounds: how much does the father's income exceed that of his son?

When the multiplier consists of more figures than one, and is greater than 12, the following rule must be observed.

Set down the multiplicand, and place under it the multiplier, so that units are under units, tens under tens, &c.

Multiply by the first, or units' figure of the multiplier, as in the last case; then multiply by the second figure in the same manner, setting the product below the one just obtained, and taking care to set down the first figure of this second product one place further to the left; then add together the two lines so obtained.

Thus,

$$\begin{array}{r} 214178 \\ 24 \\ \hline 856712 \\ 428356 \\ \hline 5140272 \end{array}$$

The multiplicand is here *first* multiplied by the 4, which gives a product 856712: it is then multiplied by 2, which gives a product 428356: and this product, being set down one place further to the left, has *ten* times its simple value; so that in fact we have multiplied the original number by 2 and by 10, that is, by 20, (because twice ten = twenty). When, therefore, the two lines are added together, we have *twenty times* the multiplicand + four times the multiplicand, or *twenty-four* times the multiplicand.

Exercise 25.

1. Multiply 89 by 47.
2. Multiply 895 by 798.
3. Multiply 7596344 by 10.
4. Multiply 6497565 by 11.
5. Multiply 3754963 by 12.
6. From 81417 take 37159.
7. From 279143 take 91712.

Express in figures,

8. Three thousand and twenty.
9. Five millions and sixty-four thousand.
10. Eleven millions one hundred thousand and seventy.

Express in words,

11. 370010010. 720210201. 600010902.

12. A borrowed of B at different times the following sums of money:—1674*l.*, 2711*l.*, 24*l.*, 18188*l.* He repaid him at different times 7650*l.*, 81*l.*, 6100*l.*, 200*l.*, 8566*l.* How much of this debt remains unpaid?

If there are more figures in the multiplier, the work is carried on in the same way, always multiplying by each figure in turn, and setting down the *first figure* of each product *one place further to the left*, than the first figure of the line immediately above it; that is to say, immediately under the figure of the multiplier which is being used.

Thus,

$$\begin{array}{r} 21417 \\ 234 \\ \hline 85668 \\ 64251 \\ 42834 \\ \hline 5011578 \\ \hline\hline \end{array}$$

Here we have in the first line of multiplication 4 times the sum of the multiplicand; in the second line we have 3 times that sum increased by its position *tenfold*, or 30 times the sum of the multiplicand: and in the third line we have twice the sum of the multiplicand, increased by its position an hundredfold, or 200 times the sum of the multiplicand: so that, when we add the three lines together, we have 200 times + 30 times + 4 times the sum of the multiplicand; that is, two hundred and thirty-four times the multiplicand.

Observe. The first figure in each product is *directly under the figure* by which the line is multiplied, in order to obtain that product.

Exercise 26.

1. Multiply 21415 by 13.
2. Multiply 361514 by 24.
3. Multiply 271416 by 36.
4. Multiply 62714 by 42.
5. Multiply 31426 by 63.
6. Add 47697, 581, 432749, 888, 7777, 1234567.

Exercise 27.

1. Multiply 81214 by 81.
2. Multiply 2717150 by 324.

3. Multiply 9121502 by 87.
4. Multiply 8121 by 6743.
5. What is the difference between 1000000 and 999999?
6. What is the difference between 1010101 and 909090?

Exercise 28.

1. Multiply 9716154 by 57216.
2. Multiply 61402015 by 3214.
3. Multiply 1047014259 by 51413.
4. Multiply 271795271468 by 3714.
5. Why is the first figure in each product one place to the left of the first figure in the line above it?
6. When the product of any figures is between 20 and 30, why do you carry only 2?
7. If the product were one hundred and eight, what would you carry? and what if it were one hundred and twenty-five?

When a cipher, or 0, occurs in the multiplier, it may be neglected; but care must be taken to place the first figure of the next line *two places* to the left, so that it may still be under the figure which is being used as a multiplier.

1.

$$\begin{array}{r} 2147\\ 205\\ \hline 10735\\ 0000\\ 4294\\ \hline 440135 \end{array}$$

2.

$$\begin{array}{r} 2147\\ 205\\ \hline 10735\\ 4294\\ \hline 440135 \end{array}$$

3.

$$\begin{array}{r} 2147\\ 205\\ \hline 10735\\ 42940\\ \hline 440135 \end{array}$$

Here we have in the first example a line of ciphers placed to correspond with the cipher by which we multiply: in the second example the ciphers are omitted altogether, and the first figure of the next product is set down two places to the left, and under its own figure in the multiplier; in the third example one cipher is set down, and then the next figure of the multiplier is proceeded with in the same line. This latter method is the most common: it is also the *best*, because the cipher being set down ensures our putting the figures of the next product in their right places. The answers in each case are the same.

Exercise 29.

1. Multiply 214167 by 2104.
2. Multiply 814126 by 60705.
3. Multiply 9141472 by 2014.
4. Multiply 614170275 by 50076.
5. Multiply 275 by 20.
6. Multiply 6759 by 1200.

DIVISION.

Simple Division is the method of finding how often one number is contained in another number of the same denomination; or by what number the former must be multiplied to become equal to the latter.

The number to be divided is called the DIVIDEND.

The number by which you divide is called the DIVISOR.

If the dividend does not contain the divisor an exact number of times, the quantity which remains over is called the REMAINDER.

The result obtained (or answer) is called the QUOTIENT.

DIVISION TABLE.

2 in	4,	Twice	3 in	6,	Twice	4 in	8,	Twice
2 ..	6,	3 times	3 ..	9,	3times	4 ..	12,	3 times
2 ..	8,	4 ..	3 ..	12,	4 ..	4 ..	16,	4 ..
2 ..	10,	5 ..	3 ..	15,	5 ..	4 ..	20,	5 ..
2 ..	12,	6 ..	3 ..	18,	6 ..	4 ..	24,	6 ..
2 ..	14,	7 ..	3 ..	21,	7 ..	4 ..	28,	7 ..
2 ..	16,	8 ..	3 ..	24,	8 ..	4 ..	32,	8 ..
2 ..	18,	9 ..	3 ..	27,	9 ..	4 ..	36,	9 ..
2 ..	20,	10 ..	3 ..	30,	10 ..	4 ..	40,	10 ..
2 ..	22,	11 ..	3 ..	33,	11 ..	4 ..	44,	11 ..
2 ..	24,	12 ..	3 ..	36,	12 ..	4 ..	48,	12 ..

5 in 10,	Twice	6 in 12,	Twice	7 in 14,	Twice
5 .. 15,	3 times	6 .. 18,	3 times	7 .. 21,	3 times
5 .. 20,	4 ..	6 .. 24,	4 ..	7 .. 28,	4 ..
5 .. 25,	5 ..	6 .. 30,	5 ..	7 .. 35,	5 ..
5 .. 30,	6 ..	6 .. 36,	6 ..	7 .. 42,	6 ..
5 .. 35,	7 ..	6 .. 42,	7 ..	7 .. 49,	7 ..
5 .. 40,	8 ..	6 .. 48,	8 ..	7 .. 56,	8 ..
5 .. 45,	9 ..	6 .. 54,	9 ..	7 .. 63,	9 ..
5 .. 50,	10 ..	6 .. 60,	10 ..	7 .. 70,	10 ..
5 .. 55,	11 ..	6 .. 66,	11 ..	7 .. 77,	11 ..
5 .. 60,	12 ..	6 .. 72,	12 ..	7 .. 84,	12 ..

8 in 16,	Twice	9 in 18,	Twice	10 in 20,	Twice
8 .. 24,	3 times	9 .. 27,	3 times	10 .. 30,	3 times
8 .. 32,	4 ..	9 .. 36,	4 ..	10 .. 40,	4 ..
8 .. 40,	5 ..	9 .. 45,	5 ..	10 .. 50,	5 ..
8 .. 48,	6 ..	9 .. 54,	6 ..	10 .. 60,	6 ..
8 .. 56,	7 ..	9 .. 63,	7 ..	10 .. 70,	7 ..
8 .. 64,	8 ..	9 .. 72,	8 ..	10 .. 80,	8 ..
8 .. 72,	9 ..	9 .. 81,	9 ..	10 .. 90,	9 ..
8 .. 80,	10 ..	9 .. 90,	10 ..	10 .. 100,	10 ..
8 .. 88,	11 ..	9 .. 99,	11 ..	10 .. 110,	11 ..
8 .. 96,	12 ..	9 .. 108,	12 ..	10 .. 120,	12 ..

11 in 22,	Twice	12 in 24,	Twice	20 in 40,	Twice
11 .. 33,	3 times	12 .. 36,	3 times	20 .. 60,	3 times
11 .. 44,	4 ..	12 .. 48,	4 ..	20 .. 80,	4 ..
11 .. 55,	5 ..	12 .. 60,	5 ..	20 .. 100,	5 ..
11 .. 66,	6 ..	12 .. 72,	6 ..	20 .. 120,	6 ..
11 .. 77,	7 ..	12 .. 84,	7 ..	20 .. 140,	7 ..
11 .. 88,	8 ..	12 .. 96,	8 ..	20 .. 160,	8 ..
11 .. 99,	9 ..	12 .. 108,	9 ..	20 .. 180,	9 ..
11 .. 110,	10 ..	12 .. 120,	10 ..	20 .. 200,	10 ..
11 .. 121,	11 ..	12 .. 132,	11 ..	20 .. 220,	11 ..
11 .. 132,	12 ..	12 .. 144,	12 ..	20 .. 240,	12 ..

To divide any number by a single figure,

Set down the number to be divided, or dividend; and a little to the left set down the figure by which you are to divide, or divisor: and draw a small curved line between them.

See how often the divisor is contained in the first figure, beginning at the left hand; or, if it be not contained in the first figure, how often it is contained in the number which the *two first figures would represent if they were not followed by any other figures.*

Set down the figure thus obtained below, for the first figure in the quotient, subtract the *product of this figure and the divisor* from that figure (or figures) which you are dividing, and consider the remainder as so many *tens* to be carried to the next figure of the dividend.

After carrying these tens, and adding them to the next figure, proceed with the number so formed in the same way; and proceed with all the figures in the same way till the whole dividend has been used.

Thus, to divide 61414 by 2.

2)21414

10707

Here we say 2 is contained in 2 once, without any remainder; we therefore set down 1 under the 2, and have nothing to carry. We then go on to the 1, but 2 is not contained in 1, we therefore set down a cipher, and carry the 1 to the next figure; but the 1, in its proper place, is equal to *ten* in the next place to the right; we therefore call it 10, and add it to the 4: we therefore have 14 to divide by 2; in this, 2 is contained exactly 7 times; we therefore set down 7, and carry nothing. In the next figure, which is 1, 2 is not contained; we therefore set down a cipher, and carry the 1 as before. We have therefore again 14 to divide by 2, which contains it exactly 7 times.

Again, to divide 157085 by 5.

5)157085

31417

Here the 5 is not contained (or will not go) in 1; we therefore take the next figure, and say 5 is contained in 15, 3 times exactly: we therefore set down 3. We then go *on to* the 7, and say 5 in 7 will go once, and 2 over; we therefore set down 1, and carry 2. We have then two *tens*, or *twenty*, to add to the next figure; and as this is a cipher, we say 5 in 20 will go exactly 4 times (because 4 times 5 = 20); we therefore set down 4, and carry nothing. The next figure is 8, in which 5 is contained once, with a remainder 3: we therefore set down 1, and carry 3 *tens* or thirty, and this, added to the 5, makes 35 to be divided; in which 5 is contained exactly 7 times.

Exercise 30.

1. Divide 16282 by 2.
2. Divide 111513 by 3.
3. Divide 522870 by 6.
4. Multiply 8121 by 3154.
5. Multiply 3717 by 309.
6. Multiply 201504 by 2706.
7. In Numeration, what would be the value of an 8 in the fourth place of the second class? of a 5 in the third place of the first class? and of a 9 in the sixth place of the first class?
8. How many ciphers would you place on the right hand of 9 to make it 9 millions? and how many to make it ninety thousand?

Let it be remembered that for every *unit*, or 1, in the remainders, we carry 10, because we are working from left to right, or from a higher to a lower place; and 1 in the place of *hundreds* = 10 in the place of *tens*: and 1 in the place of *tens* = 10 *units*.

Exercise 31.

1. Multiply 614010 by 80206.
2. Multiply 3279563 by 2005.
3. Divide 3070875 by 5.
4. Divide 24566508 by 4.
5. From 170729584 take 8969543.
6. From 827291010 take 390715973.
7. From 61025043716 take 57998762167.

When the divisor is greater than 12, and is the product of two or more numbers, each of which is less than 12, we may break this divisor into these factors or numbers, and divide by them separately; using one of them first as a divisor of the dividend, and then dividing the *quotient so produced* by the other, and so on.

Thus, to divide 96 by 16.

Sixteen = 4 times 4; we may therefore break the divisor into 4 and 4.

$$16\begin{cases}4 \\ 4\end{cases}\begin{array}{|c} 96 \\ \hline 24 \\ \hline 6 \\ \hline\hline \end{array}$$

Here we first divide 96 by 4, which gives a quotient 24; we then divide 24 by 4, which gives a quotient 6. And it appears that 6 is the fourth part of 24, and 24 is the fourth part of 96; so that 6 is the *fourth part of the fourth part of* 96; or a *quarter of a quarter* of 96: that is to say, the sixteenth part of 96.

If the student will multiply together 16 and 6, he will find that this method gives a correct answer.

Exercise 32.

1. Divide 16488 by 24.
2. Divide 126216 by 72.
3. Divide 2633988 by 42.
4. Divide 6578334 by 81.
5. Divide 1979838 by 63.
6. Divide 9770976 by 36.
7. A traveller completed a journey of 3000 miles in 12 days; how far did he travel daily?
8. Having a journey of 456 miles to make, I travelled for three days at the rate of 106 miles daily: how much of my journey then remained to be completed?
9. Why do you borrow *ten* in subtraction, and only carry *one?*

When there are more figures than one in the divisor, the work must be thus performed:

Set down the divisor and dividend as before, with a line between them; draw also a line at the opposite side of the dividend, and beyond it place the figures of the quotient in order, as they are found.

Ascertain how many times the divisor will go in as many figures of the dividend as will just make a number greater than the divisor, and set down the figure thus obtained for the first figure in the quotient. Multiply the divisor by this figure, and having placed the product under the figures, already used, of the dividend, subtract the lower line from the upper, and place the remainder below.

At the right hand of this remainder place the next figure of the dividend (or bring down the next figure), ascertain how many times the divisor is contained in this new number, and place the figure so found to the right of the one found before, for the second figure in the quotient; multiply the divisor by this figure, and place the product under the last remainder, which you have just increased by bringing down a figure from the dividend, and subtract as before: bring down the next figure of the dividend, and proceed in the same way, until all the figures of the dividend have been used.

Thus, to divide 18078368 by 253.

```
253)18078368(71456
    1771
    ----
      368
      253
      ----
      1153
      1012
      ----
       1416
       1265
       ----
        1518
        1518
        ----
        ....
        ====
```

Here we find that the divisor 253, though consisting only of 3 figures, is not contained in 180, the three first figures of the dividend; we therefore say 253 in 1807;

this will go 7 times: we therefore put down 7 for the first figure of the quotient: then we multiply 253 by 7, and set down the product 1771 under 1807, and subtract it. The remainder (which must always be less than the divisor) is 36: we then bring down the next figure of the dividend, which is 8, and place it at the right of the remainder, and say, 253 in 368; this will go once only; we therefore put 1 for the next figure in the quotient, and, setting down 253 under 368, subtract as before. The remainder this time is 115: we then bring down the 3, and say, 253 in 1153: this will go 4 times; we therefore multiply 253 by 4, and set down the product 1012 under 1153, and subtract it. The remainder is now 141; we then bring down the 6, and say, 253 in 1416; this will go 5 times; we therefore multiply 253 by 5, and set down the product 1265 under 1416, and subtract it. The remainder is 151: we then bring down the 8, and say, 253 in 1518: this will go 6 times: we multiply therefore 253 by 6, and write down the product 1518 under 1518, and have no remainder.

To get some idea of *how many times the divisor is contained in the dividend* at any stage of the work, we try how often the first figure of the divisor is contained in the first figure, or in the two first figures (if necessary) of that part of the dividend. Thus, to find how often 253 will go in 1807, we say, 2 in 18; this would go exactly 9 times; but, in multiplying by 9, we should have 4 to carry from the product of 5 and 9, which would make the quantity to be subtracted much larger than the quantity from which it would have to be subtracted; and, as this cannot be done, we must have a smaller figure for the first in the quotient: *i. e. it will not go* 9 *times.* If we were to try an 8, we should have 2 into 8 (=16) for the product; but, as 5 times 8 gives 40, we should still have 4 to carry to the 16 from the preceding figure, which would make it more than 18; and the quantity to be subtracted would again be too large. It will therefore only go 7 times.

Observe, therefore, that in trying to ascertain how often the divisor is contained in the dividend by this trial of the first figure, *considerable allowance* must be made for having to *carry* in multiplying the whole divisor.

If the remainder at any stage of the work is greater than the divisor, it is a sign that the figure in the quotient is too small, and that the divisor would have gone once more.

Thus, 253)18078368(6
1518
———
289

The remainder here is 289, which is greater than 253; it is therefore evident that 253 is contained at least once more than six times in 1807; or that the first figure of the quotient should have been 7 and not 6.

Exercise 33.

1. Divide 278395 by 13.
2. Divide 8676336 by 24.
3. Divide 1979838 by 63.
4. Divide 6513016 by 8.
5. Divide 1927559286 by 9.
6. Divide 6578334 by 81.
7. What is the difference between 81472 and 314159?
8. What is the difference between 214168 and 13121?
9. Three persons entered into partnership; A had 1206*l.*; B had twice as much as A, and C had as much as A and B together; how much had B and C, and what was the whole capital of the party?

Exercise 34.

1. Divide 88035660 by 324.
2. Divide 54759903 by 6743.
3. Divide 450607368 by 2104.
4. Divide 18410924608 by 2014.
5. Divide 5458714830 by 60705.
6. Multiply 31426 by 63.
7. Multiply 913876 by 8759.
8. A gentleman divided his estate equally between his 6 children: it yielded 4962*l.* yearly; what was the income of each child?

Sometimes the remainder is so small that, even when the next figure is brought down, the divisor is not contained in the dividend; in that case we must put a cipher for the next figure in the quotient, and bring down another figure to the dividend.

```
2146)1088022(507
     10730
     -----
       15022
       15022
       -----
       .....
       =====
```

The remainder after the first subtraction is 150; and when 2 is brought down to it, it is 1502; which is still much smaller than the divisor; we therefore put a cipher in the quotient, and bring down the next figure, which is also a 2: we have now 15022 for the dividend, in which the divisor is contained exactly 7 times.

Remember, then, that there must always be *either a significant figure or a cipher* in the quotient, for every figure which is brought down.

Exercise 35.

1. Divide 793570674 by 87.
2. Divide 880356600 by 324.
3. Divide 1713222977320 by 8.
4. Divide 517550 by 25.
5. Divide 3425928 by 57.
6. A gentleman gave to A 256*l.*: to B half as much as to A: to C twice as much: and to D three times as much: what did he give amongst them?
7. From thirteen millions take two millions eight thousand and twenty.
8. From the number represented by a 5 in the second place of the second class in the Numeration Table, take the number represented by a 9 in the sixth place of the first class, and a 4 in the second place of the first class.

Exercise 36.

1. Divide 47040968 by 67.
2. Divide 197346076210 by 3214.
3. Divide 53830144097967 by 51413.
4. Divide 2633988 by 42.

5. Add together three millions: one million four hundred thousand: eight hundred and sixty thousand and forty: one hundred and twenty-six thousand and eighty-two: eleven millions and fourteen: three thousand one hundred: eight thousand and four.

6. Add 240 to the difference between six millions and five millions eight hundred thousand and one.

When the divisor is not contained an exact number of times in the dividend, the last remainder must be set down at the right of the quotient, with the divisor placed under it.

$$\begin{array}{ll}
\text{As,} & 24)8676349(361514\frac{13}{24} \\
& \underline{72} \\
& 147 \\
& \underline{144} \\
& 36 \\
& \underline{24} \\
& 123 \\
& \underline{120} \\
& 34 \\
& \underline{24} \\
& 109 \\
& \underline{96} \\
& 13
\end{array}$$

Exercise 37.

1. Divide 28275568 by 36.
2. Divide 21278211 by 45.
3. Divide 998824405 by 108.
4. Divide 1234567890 by 7.
5. In addition, why do you only carry *one* for every *ten* in the sum?
6. When you borrow ten in subtraction, from what do you borrow it, and how do you make compensation?

Exercise 38.

1. Divide 19902672 by 324.
2. Divide 1471182327 by 4761.
3. Divide 127400818 by 1403.
4. Multiply 20702 by 25
5. Multiply 60104 by 57.
6. Multiply 90806 by 1403.

When there are any ciphers at the right hand of the divisor, the work may be shortened.

Cut off the ciphers, and cut off the same number of figures from the dividend: then divide the remaining figures according to the general rule, and to the remainder (if there is any) bring down the figures which were cut off from the dividend, for the true remainder.

Thus, 36|00)97709|76(2714 $\frac{576}{3600}$

$$\begin{array}{r}
72 \\
\hline
257 \\
252 \\
\hline
50 \\
36 \\
\hline
149 \\
144 \\
\hline
576 \\
\hline\hline
\end{array}$$

or, 3600)9770976(2714 $\frac{576}{3600}$

$$\begin{array}{r}
7200 \\
\hline
25709 \\
25200 \\
\hline
5097 \\
3600 \\
\hline
14976 \\
14400 \\
\hline
576 \\
\hline\hline
\end{array}$$

Here the sum is worked, first, by cutting off the ciphers according to the rule just given; and secondly, without cutting them off, and it will be seen that the result is the same in each case.

By cutting off the two figures in the dividend we have, in fact, already divided by 100: and as $3600 = 36 \times 100$, we have only to divide the remaining part of the dividend by 36.

The remaining 5 really belongs to the place of hundreds, because the dividends had previously been divided by 100.

Exercise 39.

1. Divide 270412 by 20.
2. Divide 9704165 by 300.
3. Divide 332064000 by 461200.
4. Divide 57297030800 by 70300
5. Express in writing, 10009001, 2790003, 68170402.
6. What is the difference between three millions, and three hundred thousand?
7. What is the difference between ninety-seven millions, and nine millions eighty-four thousand and six?

Exercise 40.

1. Divide 564397683450 by 70050.
2. Divide 370714156 by 400.
3. Divide six millions three hundred thousand by seven hundred thousand.
4. Divide one hundred and forty-four millions by twelve thousand.
5. From 8141765432, take 971416523.
6. From 67171405, take 8147296.
7. What is the difference between 9876473 and 60070052?

COMPOUND ADDITION

Is the method of collecting into one sum several quantities of different denominations.

By this rule different sums of money are added together: also different weights, measures, &c.

TABLES OF MONEY.

	Marked		Marked
Farthing	¼	4 Farthings are 1 Penny	*d.*
Halfpenny	½	12 Pence .. 1 Shilling	*s.*
3 *Farthings*	¾	20 Shillings .. 1 Pound	£.

Farthings	*d.*	*s.*	£.
4 =	1		
48 =	12 =	1	
960 =	240 =	20 =	1

Pence. *d.*		*s.*	*d.*	Pence. *d.*		*s.*	*d.*	Shillings. *s.*		£.	*s.*
20	are	1	8	100	are	8	4	20	are	1	0
24		2	0	108		9	0	30		1	10
30		2	6	110		9	2	40		2	0
36		3	0	120		10	0	50		2	10
40		3	4	130		10	10	60		3	0
48		4	0	132		11	0	70		3	10
50		4	2	140		11	8	80		4	0
60		5	0	144		12	0	90		4	10
70		5	10	150		12	6	100		5	0
72		6	0	156		13	0	110		5	10
80		6	8	160		13	4	120		6	0
84		7	0	168		14	0	130		6	10
90		7	6	170		14	2	140		7	0
96		8	0	180		15	0	150		7	10

Place the different numbers, which are to be added, under each other, in such a manner that each denomination will form a separate column, *i. e.* pounds must be under pounds, shillings under shillings, and pence under pence, when sums of money are to be added: and pounds under pounds, ounces under ounces, &c., when such weights are to be added. And in *each denomination* units must be under units, tens under tens, &c.

Begin with the lowest denomination, and add the column as in Simple Addition: see how many units of the next higher denomination are contained in the sum, and set down the remainder.

Carry these units to the next higher denomination, and add them to that column: and when the sum of that column so increased is found, see how many units of the next higher denomination are contained in it: set down the remainder, and proceed as before. Do this in succession with every denomination.

	£.	*s.*	*d.*		£.	*s.*	*d.*
Thus add,	22	11	10	to	23	15	4

£.	*s.*	*d.*
22	11	10
23	15	4
46	7	2

Here we first add together the pence, which amount to 14; but 14 pence = 1 shilling and 2 pence; we therefore set down 2 for the pence, and carry 1 to the column of shillings. Then 1 shilling with 15, and 11 shillings make 27 shillings. This is equal to 1 pound and 7 shillings; we therefore set down 7 for the shillings, and carry 1 to the column of pounds. And 1, 23, and 22, added together, make 46 for the pounds, and the whole sum is 46*l.* 7*s.* 2*d.*

Exercise 41.

1. Add together 24*l.* 11*s.* 3*d.* and 47*l.* 3*s.* 10*d.*
2. Add together 127*l.* 9*s.* 9*d.* and 69*l.* 10*s.* 3*d.*
3. Add together 1246*l* 14*s.* 7*d.* and 374*l.* 19*s.* 11*d.*
4. Multiply 2141024 by 37914.
5. Multiply 871760 by 2074.
6. From 6907021 take 3198234.
7. From 8169003 take 8079186.

Exercise 42.

1. Add together 241*l.* 4*s.* 6*d.* and 385*l.* 13*s.* 11½*d.*
2. Add together 843*l.* 14*s.* 9½*d.* and 747*l.* 14*s.* 6¾*d.*
3. Add together 369*l.* 8*s.* 6½*d.* and 464*l.* 19*s.* 10¼*d.*
4. Add together 8141*l.* 11*s.* 3*d.* and 6284*l.* 13*s.* 3½*d.*
5. From 4717016 take 62479.
6. Multiply 9716532 by 92704.

The same method is to be pursued for any number of sums that are to be added, *i. e.* however many lines may have to be added together.

	£.	*s.*	*d.*
Add,	124	3	$7\frac{1}{4}$
	36	4	11
	195	7	$6\frac{1}{2}$
	87	16	4
	960	12	$10\frac{3}{4}$
	1404	5	$3\frac{1}{2}$

Here we have first to add together 3 farthings, 1 halfpenny (or 2 farthings), and 1 farthing; which amount to 6 farthings, or 1 penny and 1 halfpenny: we therefore set down 1 halfpenny ($\frac{1}{2}$) and carry 1 to the pence. The pence column, with the 1 which we carried, amounts to 39 pence, or 3 shillings and 3 pence; we therefore set down 3 for the pence, and carry 3 to the shillings. The shillings, with the 3 which we carry, amount to 45 shillings, or 2 pounds 5 shillings; we therefore set down 5 for the shillings, and carry 2 to the pounds. The pounds, with the 2 which we carry, amount to 1404. And the whole amount is 1404*l.* 5*s.* $3\frac{1}{2}$*d.*

To find out what to carry and what to set down, we must divide the sum of each denomination by the number of units in that denomination, which make one unit in the next higher denomination, and *set down the* REMAINDER, and *carry the* QUOTIENT.

Thus the sum of the farthings must be divided by 4, because 4 farthings = 1 penny. The sum of the pence must be divided by 12, because 12 pence = 1 shilling. And the sum of the shillings must be divided by 20, because 20 shillings = 1 pound.

4)6	12)39	2,0)4,5
1 . 2	3 . 3	2 . 5

The farthings in the last example amounted to 6, which divided by 4, gives a quotient 1, and a remainder 2; we therefore set down 2 and carried 1. The pence amounted to 39, which divided by 12, gives a quotient 3, and a remainder 3: we therefore set down 3 and carried 3. The shillings amounted to 45, which divided by 20, gives a quotient 2, and a remainder 5: we therefore set down 5 and carried 2.

Exercise 43.

		£.	*s.*	*d.*
Add,	1.	241	12	7½
		327	18	6¼
		97	14	1½
		123	13	6¼

	£.	*s.*	*d.*		£.	*s.*	*d.*
2.	196	14	1	3.	2476	4	6¼
	379	16	10½		193	2	11
	24	11	6¾		29	13	7½
	3226	6	1¼		34710	16	8¾

4. Divide twenty-six millions four hundred and six thousand three hundred and nine, by five hundred and six.

5. Divide three hundred and seventy-four millions thirty-six thousand and twenty-eight, by six hundred and eleven.

6. How do you ascertain what to carry in Compound Addition?

7. When you have divided the sum of any denomination by its proper divisor, *which* do you set down, the *quotient* or the *remainder?* and which do you carry?

8. If the pence amounted to 60, what should you set down? and carry?

An easy way of proving whether the sum is correctly added is to omit the upper line, and add together the remaining ones; and then to add together the result so obtained and the upper line. And if this last amount corresponds with the amount originally found, the work may be considered to be correct.

This method of proof is similar to that given in Simple Addition.

As good a way, however, as any, is to go over the work a second time, beginning at the top instead of the bottom of each column; and if the amount so obtained corresponds with the amount obtained in the ordinary way, the work may be considered to be right.

Exercise 44.

In this exercise let the answers be proved.

Add together, 1.

£.	s.	d.
24147	11	4
81274	9	6¼
2714	6	7½
27	4	11¾
46179	9	10½
6147	18	1¾

	£.	s.	d.		£.	s.	d.
2.	8141427	9	4¼	3.	25476	0	4¼
	27140	6	3½		17214	18	3
	9123	4	5¾		96426	7	0
	6130271	13	11		2704	13	10¼
	864702	19	1½		986	12	3¾
	7141426	14	7		721416	15	0

4. Divide 60714025 by 2146.
5. Divide 312807154 by 9804.
6. Divide 34056572543 by 345.
7. What would you carry to the shillings if the pence amounted to 58?
8. What would you carry to the pounds if the shillings amounted to 106?

Instead of pounds, shillings, and pence, we often have to add weights, as hundredweights, pounds, ounces, &c.: or portions of time, as years, months, days, &c.

In these cases the work must be done in the same manner, and according to the same rule. The only difference will be that we shall have to use *different divisors* in order to ascertain what to set down and what to carry.

And these divisors will always be the *number of units* in the denomination added, which make *one unit* in the next higher denomination of the sum.

Thus, if we are adding days, we must divide by 7, in order to carry to weeks; and if we are adding weeks, we *must divide* by 4, in order to carry to months.

Or, if we are adding ounces, we must divide by 16, in order to carry to pounds; and if we are adding pounds, we must divide by 28, in order to carry to quarters, or by 14, in order to carry to stones; as will be seen by the following tables; which should all be learned by rote.

TIME.

60 Seconds .... make .... 1 Minute.
60 Minutes.............. 1 Hour.
24 Hours .... 1 Day.
7 Days................. 1 Week.
4 Weeks .... 1 Month.
12 Calendar Months }
52 Weeks, and one day or } 1 Common Year.
365 Days.............. }
366 Days................. 1 Leap Year.
100 Years .... 1 Century.

Exercise 45.

Add, 1.

mo.	wk.	day.
7	3	4
11	2	6
9	1	5
10	3	1

2.

years.	mo.	wk.	ds.
240	11	3	2
174	10	1	6
197	4	2	3
120	9	2	4

3.

mo.	wk.	ds.	hrs.
1	3	5	11
12	1	4	8
6	2	6	9
8	3	2	7

4. Add together 120*l.* 11*s.* 4½*d.*, 74*l.* 9*s.* 3*d.*, 175*l.* 18*s.* 7¾., and 59*l.* 14*s.* 6¼*d.*

5. Add together 3 weeks, 6 days, 11 hours, 40 minutes: 8 weeks, 4 days, 5 hours, 59 minutes: 7 weeks, 5 days, 16 hours, 43 minutes: and 11 weeks, 3 days, 18 hours, 39 minutes.

6. A father left the following sums to his 4 children:—To the first, 20,000*l.*; to the second, 18,400*l.* 11*s.* 6*d.*; to the third, 11,563*l.* 9*s.* 6*d.*; and to the fourth, 9045*l.* 10*s.*, what did he leave in all?

AVOIRDUPOISE WEIGHT.

		Marked
16 Drams........make	1 Ounce........	*dr. oz.*
16 Ounces	1 Pound........	*lb.*
14 Pounds	1 Stone	*st.*
28 Pounds	1 Quarter	*qr.*
4 Quarters, or 112 lbs.	1 Hund. weight..	*cwt.*
20 Hundred Weight....	1 Ton..........	*ton.*

By this table all things of a coarse nature, as groceries, &c., are weighed: and all metals except gold, silver, &c.

Exercise 46.

Add together,

	cwt.	qrs.	lb.
1.	27	2	13
	126	3	9
	240	1	11
	97	3	12

	cwt.	qrs.	lb.	oz.
2.	27	2	1	12
	150	3	11	15
	75	1	9	14
	63	2	7	10

	cwt.	qrs.	lb.	oz.
3.	1527	2	27	9
	274	1	19	3
	1842	2	11	10
	3924	1	18	4

	day.	h.	m.	sec.
4.	14	11	40	57
	15	20	39	6
	18	13	47	53
	10	1	3	48

	years.	mo.	w.	d.	h.	min.	sec.
5.	121	3	2	2	16	43	51[1]
	540	11	3	6	12	28	9
	327	9	2	4	19	52	39
	1464	11	3	5	23	36	42

6. Divide 4121859220 by 3700.

7. From 8121795432 take 81414567.

8. What is the difference between 3909295 and 912100071?

9. By what do you divide to ascertain what to carry from hours to days, and why?

TROY WEIGHT.

		Marked
24 Grains ...... make	1 Pennyweight.	*gr. dwt.*
20 Pennyweights	1 Ounce......	*oz.*
12 Ounces	1 Pound......	*lb.*

---

[1] 13 months, of 4 weeks each, making one year.

By this table, gold, silver, jewels, and most things of a costly nature are weighed.

Exercise 47.

Add together,

1.

lb.	oz.	dwt.	gr.
14	11	18	23
210	8	16	21
87	4	9	3
123	7	19	22

2.

£.	s.	d.
113	11	4
217	9	3½
3214	8	11¾
4127	19	1½

3.

lb.	oz.	dwt.	gr.
12	9	8	4
124	8	19	22
339	11	4	16
127	9	17	21

4.

day.	h.	min.	sec.
12	16	30	47
15	11	29	34
18	4	36	41
21	14	7	15
92	19	54	57

5.

oz.	dwt.	grs.
9	4	1
11	19	16
12	14	23
14	17	19
33	5	7

6.

lb.	oz.	dwt.
160	3	4
237	11	13
963	10	8
347	9	15
6	4	18

7. What sum will discharge the following bills:—Grocer's, 7*l.* 14*s.* 3*d.*; Baker's, 9*l.* 7*s.* 6½*d.*; Draper's, 1*l.* 4*s.* 7½*d.*; Carpenter's, 10*l.* 12*s.* 2¼*d.*?

8. Ten pounds were divided between A, B, C, and D: A had 1*l.* 10*s.*: B had 3*l.* 19*s.* 6*d.*: C had 3*l.*: what had D?

APOTHECARIES' WEIGHT.

		Marked
20 Grains (gr.)....make	1 Scruple......	℈
3 Scruples	1 Dram	ʒ
8 Drams	1 Ounce	℥
12 Ounces............	1 Pound	℔.

Exercise 48.

Add,

	lb.	℥	ʒ	℈	gr.
1.	12	11	6	2	18
	14	6	5	1	6
	16	3	7	0	19
	24	10	0	2	12
	36	0	4	0	6

	lb.	℥	ʒ	℈	gr.
2.	124	8	3	2	15
	380	10	6	1	0
	24	9	7	0	16
	130	3	2	1	14
	240	6	5	2	19

	lb.	℥	ʒ
3.	24	5	1
	13	2	1
	18	11	6
	24	7	2
	16	4	4

	cwt.	qrs.	lb.	oz.
4.	127	2	24	3
	94	3	5	11
	349	0	14	15
	7	2	13	8
	16	3	0	12

	£.	s.	d.
5.	1364	11	3
	393	18	4½
	25	9	1
	0	3	11½
	4614	8	4¾

	hrs.	min.	sec.
6.	12	15	37
	10	4	53
	95	18	12
	5	39	23
	16	57	14

DRY MEASURE.

		Marked
2 Pints .... make	1 Quart	*pts. qts.*
4 Quarts	1 Gallon	*gal.*
2 Gallons	1 Peck	*pk.*
4 Pecks	1 Bushel	*bu.*
2 Bushels	1 Strike	*strike.*
4 Bushels	1 Coomb, or Sack ..	*coomb, sa.*
8 Bushels	1 Quarter	*qr.*
4 Quarters	1 Chaldron	*chal.*
5 Quarters	1 Load	*load.*
10 Quarters	1 Last	*last.*

By this table, corn, seeds, roots, salt, and all dry articles are measured.

Exercise 49.

Add together,

	qr.	bu.	pk.	gal.
1.	36	7	3	1
	94	4	2	0
	136	5	1	1
	49	6	3	0
	184	7	0	1
	43	3	2	0

	qr.	bu.	pk.	qts.
2.	120	6	2	6
	25	1	2	4
	439	5	1	7
	87	4	0	1
	133	7	3	5
	84	4	1	6

	£.	s.	d.
3.	9374	14	4¾
	8712	3	11½
	241	4	7
	8779	13	6¼
	705	15	2¾
	121	8	0
	3742	4	7½
	985	12	4¾

	lb.	℥	ʒ	℈	gr.
4.	14	11	2	1	19
	15	8	7	1	14
	124	10	6	2	11
	97	4	6	0	18
	432	9	4	2	14
	92	10	7	1	12

	cwt.	qr.	lb.	oz.	drs.
5.	1432	3	24	14	14
	411	2	18	12	9
	9157	1	23	15	6
	841	0	11	3	4
	12804	3	8	9	7
	2579	1	14	14	13
	863	2	7	15	4

	qr.	bu.	pk.	qts.
6.	147	7	3	7
	97	6	1	6
	254	4	0	5
	876	3	2	4
	711	6	1	6
	899	3	0	2
	254	5	3	7

7. What time elapsed between the call of Abraham (B.C. 1921), and the visit of Augustin to England (A.D. 596)?

8. What time elapsed between the confusion of tongues (B.C. 2233), and the gift of tongues (A.D. 33)?

WINE MEASURE.

		Marked
4 Gills ......make	1 Pint............	*pt.*
2 Pints	1 Quart	*pts. qts.*
4 Quarts	1 Gallon..........	*gal.*
10 Gallons	1 Anker of Brandy ..	*ank.*
18 Gallons	1 Rundlet	*run.*
42 Gallons	1 Tierce	*tierce.*
2 Tierces, or 84 Gal.	1 Puncheon	*pun.*
63 Gallons	1 Hogshead	*hhd.*
2 Hogsheads	1 Pipe or Butt	*p.* or *butt.*
2 Pipes, or 4 Hogsh.	1 Tun............	*tun.*

Exercise 50.

Add together,

	hhd.	gal.	qts.	pts.
1.	27	53	2	1
	96	40	1	0
	87	36	3	1
	77	54	0	0
	125	10	1	0
	193	60	2	1

	pipe.	hhd.	gal.	qts.
2.	27	1	60	3
	97	0	36	1
	125	1	61	0
	84	0	45	2
	163	0	27	3
	99	1	54	2

	tun.	hhd.	gal.	qts.
3.	27	3	60	3
	156	2	29	1
	973	1	54	2
	169	2	19	3
	29	0	56	2
	171	2	47	1

	pun.	trc.	gal.	qts.
4.	27	1	40	3
	19	0	37	2
	26	1	41	0
	39	0	27	3
	85	1	19	1
	67	0	11	2

	lb.	℥	ʒ	℈
5.	25	8	7	2
	91	11	5	1
	29	7	3	2
	35	9	6	1
	43	8	6	0
	95	1	3	2

	lb.	oz.	dwt.	gr.
6.	11	3	4	1
	19	11	19	23
	34	5	17	20
	56	4	13	19
	25	8	14	23
	19	1	11	5

7. Divide 25613634 by 3154.
8. Divide 450607368 by 2104.

ALE AND BEER MEASURE.

			Marked
2 Pints	make	1 Quart	*pts. qts.*
4 Quarts		1 Gallon	*gal.*
9 Gallons		1 Firkin	*fir.*
2 Firkins		1 Kilderkin	*kil.*
4 Firkins, or 2 Kilderkins		1 Barrel	*bar.*
54 Gallons		1 Hogshead	*hhd.*
2 Barrels		1 Puncheon	*pun.*
3 Barrels, or 2 Hogsheads		1 Butt	*butt.*
2 Butts, or 216 Gallons		1 Tun	*tun.*

Exercise 51.

Add together,

	butt.	hhd.	gal.	qts.
1.	1	1	29	3
		1	51	2
	1	0	26	1
	1	1	18	2
	1	0	47	3
		1	19	2

	hhd.	fir.	gal.	qts.
2.	46	2	7	3
	33	0	8	2
	63	1	6	1
	94	2	4	3
	87	1	0	2
	56	1	6	3

	£.	s.	d.
3.	127	19	11½
	342	14	3¼
	864	11	2½
	976	3	6
	14	11	8½
	716	18	4¾

	ton.	cwt.	lb.	oz.
4.	67	18	110	14
	96	12	87	12
	125	9	94	3
	347	15	108	10
	947	13	24	5
	856	9	2	11

	lb.	℥	ʒ	℈
5.	27	11	7	2
	96	8	3	1
	57	9	6	2
	84	1	3	0
	97	4	0	1
	125	8	5	3

	lb.	oz.	dwt.	gr.
6.	26	11	19	23
	95	10	14	13
	84	1	11	3
	19	2	9	15
	127	8	16	21
	93	9	17	5

7. How many years intervened between the Conquest (1066), and the accession of George the Third (1760)?

8. How many years intervened between the invention of gunpowder (1344), and the gunpowder conspiracy (1605)?

9. Divide 545269824 by 2706.

10. Multiply 2717150 by 324.

CLOTH MEASURE.

			Marked
2¼ Inches	make	1 Nail	*in. nl.*
4 Nails		1 Quarter of a Yard	*qrs.*
4 Quarters		1 Yard	*yd.*
5 Quarters		1 Ell	*E. E.*
3 Quarters		1 Flemish Ell	*Fl. E.*

Exercise 52.

Add together,

1.

E. E.	qrs.	nl.	in.
11	4	3	2
18	3	2	0
17	1	2	1
256	3	1	0
29	4	3	2
84	4	1	1

2.

Fl. E.	qrs.	nl.
27	2	3
64	1	2
95	0	1
136	2	0
127	0	2
65	1	3

3.

yds.	qrs.	nl.	in.
184	3	3	2
161	2	1	0
95	1	0	1
84	3	2	0
95	1	0	2
16	2	2	1

4.

£.	s.	d.
954	3	4½
847	2	1
913	1	9¼
365	14	11½
847	13	4¾
654	18	9¾

5.

ton.	cwt.	qrs.	lb.
27	18	3	26
194	11	2	16
345	4	1	18
976	13	2	11
854	11	0	19
37	2	1	4

6.

year.	mo.	d.	h.
27	11	24	19
153	8	13	2
95	3	14	1
167	10	16	13
59	4	11	3
364	5	23	23

7. A mercer bought 4 pieces of cloth: the first measured 137 yds. 3 qrs. 2 nl.; the second, 39 yds. 2 qrs. 1 nl.; the third, 256 yds. 0 qrs. 1 nl.; and the fourth, 975 yds. 2 qrs. 1 nl. 2 in.: what were their united lengths?

8. A grocer received goods by 4 vessels: the first cargo weighed 87 ton 16 cwt. 11lb.; the second, 153 ton 3 qrs. 10 oz.; the third, 93 ton 11lb. 13 oz. 12 dr.; the fourth, 194 ton 18 cwt. 23 lb. 13 dr.: what were their united weights?

LONG MEASURE.

		Marked
3 Barley-corns make	1 Inch	*bar. in.*
4 Inches	1 Hand	*hd.*
12 Inches	1 Foot	*ft.*
3 Feet	1 Yard	*yd.*
6 Feet	1 Fathom	*fth.*
5½ Yards	1 Rod or Pole	*rod, p.*
40 Poles	1 Furlong	*fur.*
8 Furlongs	1 Mile	*mile.*
3 Miles	1 League	*leag.*
69 Miles	1 Degree	*deg.*

Exercise 53.

		deg.	fur.	p.	yds.
Add together,	1.	21	7	36	4
		53	4	18	1
		75	6	14	3
		87	1	11	1
		95	4	39	2
		81	6	14	1

	miles.	fur.	rod.	ft.
2.	24	3	30	10
	187	4	11	9
	957	3	2	8
	843	7	4	13
	721	1	18	4
	929	5	24	15

	yds.	ft.	in.	bar.
3.	160	2	11	1
	904	2	8	2
	356	1	1	0
	424	0	7	1
	356	0	4	2
	29	0	6	0

4. A traveller pursued his journey for 5 successive days: on the first he went 64 miles 4 fur. 13 p.; on the second, 1 deg. 18 miles 3 fur.; on the third, 2 deg. 4 miles 13 p.; on the fourth, 2 deg. 13 miles 36 p. 3 yds.; and on the fifth, 47 miles 4 fur. 18 p.: how far did he travel?

5. Multiply 814147 by 30471.

6. Multiply 61479 by 270406.

7. Divide 24807873237 by 30471.

8. Divide 16624290474 by 61479.

COALS.

		Marked
4 Pecks make	1 Bushel	bu.
3 Bushels	1 Sack	sa.
12 Sacks, or 36 Bushels	1 Chaldron	chal.
21 Chaldron	1 Score	score.

In London, coals are sold by Avoirdupoise Weight, and

1 Sack...... weighs 224 Pounds.
10 Sacks .... make 1 Ton.

Exercise 54.

Add together,

1.

chal.	sa.	bu.
36	8	2
27	4	1
35	11	0
64	3	2
18	1	1

2.

chal.	sa.	bu.
217	9	1
314	11	2
814	10	1
316	0	1
284	9	2
614	1	0

3.

chal.	sa.	bu.	pk.
814	1	0	2
314	10	2	3
368	4	1	2
971	3	1	0
814	9	2	3
684	1	2	1

4. Having bought goods for which I paid 154*l.* 17*s.*, I was charged for packages 13*s.* 8*d.*, and for carriage 1*l.* 5*s.* 4*d.*, what was the whole cost?

5. The annual rent of a house is 29*l.* 14*s.* 2½*d.*, the land-tax 4*l.* 7*s.* 6¼*d.*, the poor's-rate 3*l.* 9*s.* 4¼*d.*, the church-rate 12*s.* 6½*d.*, what is the yearly expense?

6. In carrying from one denomination to another, how do you find out what is the proper number?

7. What divisor would you use to ascertain what to carry from pounds to hundredweights? (2) From minutes to hours? (3) From furlongs to miles?

8. To 18,414*l.* 11*s.* 6*d.*, add the difference between seven hundred thousand four hundred pounds, and half a million pounds.

9. Write in words, 8001007140210, 6900007102400.

SQUARE MEASURE.

144	sq. Inches	....make	1 sq. Foot.
9	sq. Feet	....	1 sq. Yard.
100	sq. Feet	....	1 sq. Flooring
$30\frac{1}{4}$	sq. Yards	....	1 sq. Rod, Pole, or Perch.
40	sq. Rods	....	1 sq. Rood.
4	sq. Roods	....	1 sq. Acre of Land.
640	sq. Acres	....	1 sq. Mile.

SOLID MEASURE.

1728	Inches	..........make	1 Cubic Foot.
27	Cubic Feet	....	1 Yard, or load of Earth.
40	Ft. of round Timber, or		are 1 Ton or Load.
50	Ft. of hewn Timber..		

Exercise 55.

Add,

	acre.	r.	p.
1.	380	1	34
	608	3	14
	74	3	18
	621	2	7

	acre.	r.	p.
2.	216	1	36
	917	3	21
	214	1	32
	718	3	19

	acre.	r.	p.
3.	817	2	19
	634	3	6
	915	1	19
	684	2	25

	£.	s.	d.
4.	127	8	11
	641	3	$4\frac{1}{2}$
	927	18	$10\frac{3}{4}$
	647	12	$6\frac{1}{2}$
	141	9	9

	£.	s.	d.
5.	14156	8	4
	21714	19	$2\frac{3}{4}$
	617	1	$0\frac{1}{2}$
	924	8	$3\frac{1}{2}$
	328	18	7

	chal.	bu.	pk.
6.	21	18	3
	96	32	1
	124	16	2
	965	35	1

7. What is the difference between seven hundred and four thousand and nine, and six hundred and twenty-nine thousand one hundred and eighteen?

8. What is the product of eight thousand and four, and seven thousand five hundred and three?

COMPOUND SUBTRACTION

Is the method of finding the difference between any two sums of different denominations.

The sum to be subtracted must be placed under the other, in such a manner that the parts which are of the same denomination may fall under each other, namely, pounds must be under pounds, shillings under shillings, and pence under pence when the sums to be subtracted are money; and pounds under pounds, ounces under ounces, and grains under grains, &c., when different weights have to be subtracted.

Begin with the lowest denomination, and subtract the lower sum from the upper, noting down the remainder; then proceed to the next denomination; again subtract the lower number from the upper, and note down the remainder; proceed in the same manner with each denomination until the work is completed.

Thus,	From	£245	11	$4\frac{3}{4}$
	Take	132	9	$3\frac{1}{2}$
		£113	2	$1\frac{1}{4}$

Here we first subtract 1 halfpenny (or 2 farthings) from 3 farthings, which leaves a remainder 1 farthing; this we note down. We then have to take 3 pence from 4 pence, which leaves a remainder 1 penny; this we note down. We next have to subtract 9 shillings from 11 shillings, which leaves a remainder 2 shillings to be noted down; and we then have 132 pounds to subtract from 245 pounds, which leaves a remainder 113 pounds to be noted down, and the difference between the two sums is 113*l.* 2*s.* $1\frac{1}{4}$*d.*

If in any denomination there is no significant figure in the lower line, (or subtrahend,) or if there is nothing to be subtracted, of course the sum to be noted down in that denomination is the sum which stands in the upper line.

Thus,	From	£263	18	$11\frac{1}{2}$
	Take	240	0	6
		£23	18	$5\frac{1}{2}$

Here there is nothing to be subtracted from the halfpenny in the upper line; we therefore set down 1 halfpenny as the result. There is also nothing to be subtracted from the 18 shillings in the upper line of that denomination; we therefore set down at once 18 shillings as the result. The other denominations must be subtracted as in the last example.

Exercise 56.

1. From 395*l.* 14*s.* 6*d.* take 163*l.* 7*s.* 2*d.*
2. From 647*l.* 19*s.* 4¾*d.* take 516*l.* 17*s.* 1¼*d.*
3. From 821*l.* 12*s.* 9½*d.* take 410*l.* 6*s.* 6*d.*
4. From 914*l.* 12*s.* 6*d.* take 802*l.* 8*s.* 3*d.*
5. From 1641*l.* 18*s.* 4¾*d.* take 719*l.* 16*s.* 1½*d.*
6. From 1409*l.* 11*s.* 9*d.* take 1200*l.* 4*s.* 7*d.*
7. A received from B 27*l.* 10*s.* 6*d.*; from C 146*l.* 18*s.* 1½*d.*; from D 914*l.* 11*s.* 10*d.*; from E 264*l.* 9*s.* 4¼*d.*; and from F 68*l.* 11*s.* 8¾*d.*: how much did he receive?
8. A tradesman received 4 parcels of goods: the first weighed 4 cwt. 3 qrs. 11 lb.; the second, 67 cwt. 1 qr. 16 lb.; the third, 29 cwt. 0 qr. 13 lb.; and the fourth, 96 cwt. 3 qrs. 17 lb.: what did they weigh together?

But sometimes the sum to be subtracted is larger than the sum of the same denomination in the upper line. It is evident that in this case the work cannot be performed without *borrowing.*

If therefore any number in the lower line is greater than the number which stands above it, borrow as many units in this denomination as will make one unit of the next higher denomination: add these borrowed units to those in the upper line, and from the sum subtract the number in the lower line; and note down the remainder. Then add 1 to the next denomination *in the lower line* before you subtract it.

Thus,

From	£649	1	6¼
Take	138	4	3½
	£510	17	2¾

Here we have 1 halfpenny (or 2 farthings) in the lower line to subtract from 1 farthing in the upper line: this cannot be done: we therefore *borrow* 1 penny, or 4 farthings from the pence, and say, take 2 farthings from 5 farthings: this gives a remainder 3 farthings, which we note down. Having borrowed 1 penny from the 6 pence, we have now to carry 1 as a compensation, and we therefore do not say, take 3 from 6, but take 4 from 6, and there remain 2 pence: this we note down. We then have to take 4 shillings from 1 shilling. As this cannot be done, we borrow 1 pound, or 20 shillings, and say, take 4 from 21: this leaves a remainder, 17 shillings; we therefore note down 17, and carry 1 to the pounds; so that here again we do not say, take 8 from 9, but take 9 from 9, which leaves no remainder; we therefore set down a cipher, and proceed to the next figure in the pounds.

The borrowing and carrying may be explained thus: we do not say, take 3 from 6, because we have not really 6 pence, but only 5 *pence*, from which to subtract the 3; for we *borrowed* 1 penny from the 6, and 3 taken from 5 leaves 2. But 4 from 6 also leaves 2. It is therefore immaterial, whether we consider the 6 as a 5, or the 3 as a 4; that is, whether we take 1 from the 6, and then subtract 3; or add 1 to the 3, and subtract the sum from 6. The latter method is generally adopted as being the easiest.

If in the pounds any one of the figures in the lower line is larger than the corresponding figure in the upper line, we must borrow 10 and carry 1, as in Simple Subtraction.

Thus,	From	£841	9	3½
	Take	199	4	6
		£642	4	9½

Here we have nothing to subtract from the halfpenny; we therefore set it down in the result. In the pence we have 6 to take from 3, which we cannot do: we therefore borrow 1 shilling or 12 pence, and say, take 6 from 15: this gives a remainder 9, which we note down. We have now to carry 1 to the shillings, to compensate for the 1 shilling borrowed; and we say, (not 4, but) 5 from 9, there remain *4*. We have now nothing to carry; we therefore have in the pounds to take 9 from 1. This we cannot do: we therefore borrow 10, and say 9 from 11, there remain 2; carry 1, 9 and 1 are 10; 10 from 4, we cannot; borrow 10; 10 from 14 there remain 4; carry 1; 1 and 1 are 2; 2 from 8 there remain 6; and the result is 642*l.* 4*s.* 9½*d.*

Exercise 57.

1. From 812*l.* 11*s.* 6*d.* take 617*l.* 14*s.* 7*d.*
2. From 912*l.* 8*s.* 4½*d.* take 154*l.* 15*s.* 3¾*d.*
3. From 2141*l.* 3*s.* 11½*d.* take 812*l.* 13*s.* 4½*d.*
4. From 9141*l.* 4*s.* 9¼*d.* take 632*l.* 7*s.* 8½*d.*

Add together,

	£.	s.	d.
5.	8142	13	6
	6154	11	9½
	5712	6	8¼
	9147	16	11

	year.	mo.	w.	d.
6.	8	9	3	6
	12	11	2	5
	64	8	1	4
	97	10	2	1

7. A was born on the 12th of March, 1827, and B on the 28th of January, 1834: what was the difference of their ages, in years, months, weeks, and days?

8. C was born October 27th, 1811, and D on April 4th, 1823: how much older was C than D?

9. John had 3047*l.* 8*s.* 11½*d.*; and William had 1216*l.* 13*s.* 6¾*d.*: how much more had John than William?

The same method must be pursued when the numbers to be subtracted represent *weights*, as hundredweights, quarters, pounds, &c., or *periods of time*, as months, weeks, days, &c.

The only difference in such cases will be, that we shall have to borrow different numbers, when the number in the lower line is greater than the corresponding number in the upper line.

The sum to be borrowed will in every case be the number of units in that denomination, which make one unit in the next higher denomination.

Thus, in *pence* we borrow 12, because 12 pence make 1 shilling.

In *shillings* we borrow 20, because 20 shillings make 1 pound.

And in subtracting periods of time,

In *days* we borrow 7, because 7 days make 1 week.

In *weeks* we borrow 4, because 4 weeks make 1 month.

In *months* we borrow 12, because 12 months make 1 year. So also in subtracting weights (avoirdupois).

In *ounces* we borrow 16, because 16 ounces make 1 pound.

In *pounds* we borrow 14, because 14 pounds make 1 stone, &c. &c.

And in all these cases we only carry 1, because we never borrow more than the value of *one unit* in the next higher denomination.

Exercise 58.

1. From 6 mo. 3 wks. 4 ds., take 4 mo. 2 wks. 6 ds.

2. From 127 yrs. 8 mo. 3 wks., take 98 yrs. 7 mo. 2 wks. 6 ds.

3. From 217*l.* 10*s.* take 109*l.* 15*s.* 8¾*d.*

4. My servant's wages are 12*l.* yearly, I have advanced him 5*l.* 17*s.* 6¼*d.*; what is due to him?

5. A borrowed of B 100*l.*, but returned him at different times the following sums, 21*l.* 14*s.* 6*d.*: 10*l.* 10*s.*; 21*l.*; 5*l.* 15*s.*; 18*l.* 14*s.* 9*d.*: how much remains due?

6. Having received at different times the following sums, viz , 1254*l.* 11*s.* 4*d.*; 179*l.* 14*s.* 6½*d.*; 271*l.* 15*s.* 9¼*d.*; 4801*l.* 19*s.* 11¾*d.*; 875*l.* 4*s.* 3*d.*; and 9414*l.* 12*s.* 9½*d.*: how much have I received in all?

7. Having bought articles which cost the following sums of money, viz., 130*l.* 10*s.* 6*d.*; 340*l.* 10*s.*; 27*l.* 14*s.* 6½*d.*; 79*l.* 8*s.* 4*d.*: how much remains out of 600*l.* which I had at first?

Exercise 59.

1. From 161 cwt. 1 qr. 19 lb., take 28 cwt. 3 qrs. 16 lb.

2. From 17 ton 14 cwt. 1 qr. 3 lb., take 9 ton 6 cwt. 3 qrs. 4 lb.

3. From 126 cwt. 2 qrs. 12 lb. 10 oz., take 75 cwt. 3 qrs. 16 lb. 15 oz.

4. *From* 421*l.* 13*s.* 4¾*d.*, take 305*l.* 9*s.* 8½*d.*

5. If A had 1207*l.* 14*s.* 6*d.*, and B had 816*l.* 16*s.* 9*d.*, how much more had A than B?

6. If a labourer engaged to complete a piece of work in 5 days (of 12 hours to each day), and having worked the following times, viz., 13 hrs. 45 min. 27 sec.; 9 hrs. 30 min. 48 sec.; 12 hrs. 16. min. 34 sec.; 18 hrs. 10 min. 3 sec.: completed his task, how much time had he to spare?

7. Find the amount of the following sums of money: 125*l.* 11*s.* 3*d.*; 6724*l.* 18*s.* 5½*d.*; 917*l.* 16*s.* 4*d.*; 25*l.* 11*s.* 7*d.*; 4321*l.* 1*s.* 6¾*d.*; and 640*l.* 9*s.* 11½*d.*

Exercise 60.

1. From 136 English ells, take 120 ells 1 qr. 1 nl.

2. From 360 yards, take 147 yds. 3 qrs. 3 nl.

3. From 18 lb. 6 ℥ 4 ʒ 2 ℈, take 16 lb. 8 ℥ 7 ʒ 1 ℈.

4. From 1096*l.* 14*s.* 7¾*d.* take 595*l.* 12*s.* 9½*d.*

5. What is the difference between two thousand pounds, and one thousand three hundred and twenty-four pounds eight shillings and fourpence?

6. At twenty seven minutes after 10 o'clock, how long does it want to half-past one o'clock?

7. A merchant in six voyages realized the following sums, viz., 14,147*l.* 16*s.* 5*d.*; 8121*l.* 11*s.* 3¾*d.*; 2148*l.* 13*s.* 10½*d.*; 914*l.* 8*s.* 7*d.*; 4125*l.* 16*s.* 4¾*d.*; and 13,412*l.* 9*s.* 11*d.*: how much did he realize in all?

8. A, having to perform a journey of 120 miles, completed a distance of 97 miles 7 fur. 27 p., and was taken ill and unable to proceed; how far was he from his journey's end?

COMPOUND MULTIPLICATION

Is the method of finding the amount of any *compound number* when repeated a given number of times.

It is, indeed, a short method of adding the same compound number together several times.

Thus, if we had to add 124*l.* 3*s.* 6*d.* to itself 5 times, we might do it by the rules for Compound Addition.

£.	s.	d.
124	3	6
124	3	6
124	3	6
124	3	6
124	3	6
£620	17	6

But it is more easily done by Multiplication.

Thus,

£.	s.	d.
124	3	6
		5
£620	17	6

To multiply by a single figure.

Set down the sum to be multiplied, and under the lowest denomination place the figure by which it is to be multiplied.

Multiply the lowest denomination by this figure, and see how many *units of the next higher denomination* are contained in the product; note down the *remainder*, and carry these *units*.

Then multiply the figures in the next higher denomination by the same multiplier, and to the product add the units which were to be carried from the last multiplication: again, see how many units of the next higher denomination are contained in this product; note down the *remainder*, and carry the units as before. Proceed in this manner until every denomination in the multiplicand has been thus multiplied; and the several results noted down will be the product of the whole sum.

Thus, to multiply 124*l.* 3*s.* 6*d.* by 5.

£.	s.	d.
124	3	6
		5
£620	17	6

Here we first multiply the 6 pence by 5, which gives 30 pence; but 30 pence = 2 shillings and 6 pence, we therefore set down the 6 and carry 2 to the shillings. We then multiply the 3 shillings by 5 which gives 15 shillings; to this we add the 2 shillings which we had to carry, and have on the whole 17 shillings; and, as in this sum there are no units of the next higher denomination, or pounds, we set it down, and have nothing to carry. We then proceed to the pounds, and multiplying 124 by 5, we obtain the result 620. And the whole product is 620*l.* 17*s.* 6*d.*, the same sum which was obtained by Compound Addition.

Exercise 61.

1. Multiply 71*l.* 12*s.* 6*d.* by 2.
2. Multiply 1*l.* 16*s.* 6¼*d.* by 12.
3. Multiply 128*l.* 11*s.* 6*d.* by 9.
4. Multiply 724*l.* 10*s.* 4*d.* by 8.
5. What is the cost of 9 articles at 12*l.* 7*s.* 6*d.* each?
6. What must I pay for 11 oxen at the cost of 17*l.* 10*s.* 6*d.* each?
7. What do you carry to the shillings when the pence amount to 120?
8. In Compound Subtraction, what do you borrow when the sum to be subtracted in any denomination is greater than the sum which stands above it?

If the numbers to be multiplied represent *time*, or *weights*, we proceed in the same manner, using the *tables of time*, or of *weight*, to ascertain what to carry, and what to note down in each case.

Thus to multiply 6 weeks 4 days 11 hours by 7.

w.	d.	h.
6	4	11
		7
46	3	5

Here we have 7 times 11, or 77 hours, which = 3 days and 5 hours (because 3 × 24 = 72), we therefore set down 5 for the hours, and carry 3 to the days. Then we have 7 times 4, or 28 days; and we carried 3 days; we therefore have upon the whole 31 days; this equals 4 weeks and 3 days, and we set down 3, and carry 4. We then have 7 times 6, or 42 weeks, to which we have to carry 4 weeks; so that on the whole we have 46 weeks. And the whole product is 46 weeks, 3 days, 5 hours.

Exercise 62.

1. Multiply 9 months 3 weeks 6 days by 6.
2. Multiply 1 year 11 months 2 weeks 4 days by 9.
3. Multiply 16 years 8 months 1 week by 8.
4. Multiply 120 years 7 months 3 weeks by 5.
5. John was 7 years 3 months 2 weeks old; and Henry was 4 times as old as John. How old was Henry?
6. From 256*l.* 8*s.* 7*d.*, take 113*l.* 9*s.* 10½*d.*
7. From 500*l.* take 426*l.* 1*s.* 2¼*d.*
8. A gentleman had an income of 3047*l.* 8*s.* 9*d.*, which he spent yearly; he lived, after he came into possession of it, eleven years. How much did he spend in that time?
9. A charitable person spent 647*l.* 18*s.* 4*d.* yearly in doing good, how much did he thus spend in 12 years?

If the number represent weights,

Let 24 cwt. 3 qrs. 9 lbs. 4 oz. 3 drams, be multiplied by 4.

cwt.	qrs.	lb.	oz.	dr.
24	3	9	4	3
				4
99	1	9	0	12

Here we have 3 drams to multiply by 4, which gives 12 drams. This sum does not contain an unit of the next higher denomination (or ounces), we therefore set down 12, and have nothing to carry. We then have 4 ounces to multiply by 4, which gives 16 ounces: this contains exactly one unit of the next higher denomination with nothing over (because 16 ounces = 1 pound), we therefore set down 0, and carry 1. Then multiplying 9 pounds by 4, we have 36 pounds; and we carried 1 pound; we have therefore on the whole 37 pounds: this equals 1 quarter (28 lbs.) and *9 pounds*; we therefore set down 9 for the pounds, and carry 1. In the quarters we have 3 to multiply by 4, which gives 12; and we carried 1; so that we have 13 quarters; this equals 3 hundredweight and 1 quarter; we therefore set down 1 for the quarters, and carry 3 to the hundredweights. Then multiplying 24 hundredweight by 4, we have 96 hundredweight; and we had 3 to carry; we have therefore on the whole 99 hundredweight, and the whole product is 99 cwt. 1 qr. 9 lbs. 0 oz. 12 drs.

Exercise 63.

1. Multiply 64 cwt. 3 qrs. by 5.
2. Multiply 127 cwt. 1 qr. 16 lb. by 8.
3. Multiply 1644 cwt. 0 qr. 16 lb. 8 oz. by 9.
4. A tradesman received 9 bales of goods, each weighing 22 cwt. 3 qrs. 13 lbs: what was the whole weight?
5. What is the cost of 13 pounds of tea at 4*s*. 10½*d*. per pound?
6. What is the product of 7564965 × 5003007?
7. Divide 714210 by 798.
8. From 337*l*. 17*s*. 2½*d*., take 26*l*. 12*s*. 6¼*d*.

Exercise 64.

1. Multiply 4*l*. 11*s*. 5*d*. by 9.
2. Multiply 13 cwt. 3 qrs. 11 lbs. by 10.
3. Five persons had 27*l*. 16*s*. 8¼*d*. each, how much had they together?
4. A gentleman gave to 10 poor widows three half-crowns each, how much did he give away?
5. What is the yearly amount of my servant's wages at 1*l*. 8*s*. 6¼*d*. a month?
6. A, B, C, D, entered into partnership; A with 2014*l*. 11*s*. 6*d*.; B with 3409*l*. 8*s*. 4*d*.; C with 2311*l*. 6*s*. 8*d*.: and D with 4928*l*. 3*s*. 6*d*.: what was their capital?
7. A has 456*l*. 11*s*. 6*d*.; and B has ten times as much: how much have they together?
8. Multiply 370*l*. 18*s*. 6½*d*. by 9.

When the multiplier is greater than twelve, and is itself the product of two numbers, it may be divided into those two numbers; and by multiplying the given sum by one of those numbers, and then multiplying the *product* by the *other number*, we obtain the true product of the multiplier and multiplicand.

Thus, if we have to multiply by 16, we may divide it into 4 and 4; because 4 times 4 = 16: and it is evident, that if we first multiply the given sum by 4, and the product so obtained again by 4, we shall upon the whole have 4 *times* 4 *times*, that is 16 times, the original sum.

If 20 be the multiplier, it may be divided into 5 and 4, because 5 times 4 = 20, and if we first multiply by 5, we obtain a sum equal to 5 times the original sum; and if we then multiply this result by 4, we have a result equal to 4 *times* 5 *times*, or 20 times, the original sum.

So 30 may be divided into 5 and 6; because 5 times 6 = 30; and 32 into 4 and 8; and 24 into 4 and 6; or 3 and 8, &c. &c.

Let it be required to multiply 145*l.* 4*s.* 7*d.* by 15.

The multiplier may be divided into 3 and 5, and the work done thus:—

$$\begin{array}{rrrl} \pounds. & s. & d. & \\ 145 & 4 & 7 & \\ & & 3 & \times 5 = 15 \\ \hline 435 & 13 & 9 & = 3 \text{ times.} \\ & & 5 & \\ \hline \pounds 2178 & 8 & 9 & = 15 \text{ times.} \end{array}$$

Exercise 65.

1. Multiply 1*l.* 11*s.* 6*d.* by 24.
2. Multiply 1*l.* 19*s.* 4*d.* by 28.
3. Multiply 1*l.* 18*s.* 6*d.* by 64.
4. Multiply 476*l.* 9*s.* 11*d.* by 56.
5. Multiply 1214*l.* 18*s.* 4*d.* by 72.
6. Multiply 613*l.* 8*s.* 6*d.* by 48.
7. What is the cost of 24 sheep at 2*l.* 13*s.* 6*d.* each?
8. What is the cost of 18 oxen at 17*l.* 10*s.* 6*d.* each?

Exercise 66.

1. When coals were 1*l.* 11*s.* 6*d.* a chaldron, what was *the cost of* 24 chaldron?

2. If I walk 9 miles 3 furlongs every day, how far shall I walk in a month?

3. If tea is 6*s.* 4*d.* a pound, what will be the cost of two stone?

4. What change shall I receive from a fifty pound note, if I have to pay a bill of 19*l.* 19*s.* 11¾*d.*?

5. Explain the method of multiplying by two simple figures, instead of multiplying by their product; what object is gained by it? and is the result the same?

6. Write in words the following number,
80700123404560078900.

Very frequently, however, the multiplier will not thus divide, not being the exact product of two numbers. In that case it is usual to find two numbers whose product will *nearly equal* the multiplier, and having multiplied the sum by these two numbers, according to the last rule, to add to the product the original sum *multiplied by that number which represents the* EXCESS *of the multiplier over the product of the two new multipliers.*

Thus, if we have to multiply by 17, we may take 4 and 4 for the two new multipliers, and 1 *for the excess of the multiplier over their product*, because 17 = 16 and 1, or = 4 times 4 and 1 over; and if we first multiply by 4, and then multiply the product so obtained by 4, we obtain a result equal to 4 times 4 times, or 16 times, the original sum; and then adding to this *once* the original sum, we upon the whole obtain a result equal to 17 times the original sum.

Let it be required to multiply 23*l.* 4*s.* 5*d.* by 17.

17 equals 16 + 1 or = 4 times 4 + 1.

£.	*s.*	*d.*	
23	4	5	
		4	× 4 + 1 = 17
92	17	8	= 4 times.
		4	
371	10	8	= 16 times.
23	4	5	= once.
£394	15	1	= 17 times.

But 17 also equals 15 + 2 or = 3 times 5 + 2.

We shall therefore obtain the same result if we multiply by 3, then by 5, and add *twice* the original sum (or upper line).

£.	s.	d.	
23	4	5	
		3	× 5 × 2 = 17
69	13	3	= 3 times.
		5	
348	6	3	= 15 times.
46	8	10	= twice.
£394	15	1	= 17 times.

But 17 also equals 18 — 1 (less by 1) = 3 times 6 — 1.

The same result may therefore be obtained by multiplying by 3 and 6, and subtracting the original sum.

	£.	s.	d.	
Thus,	23	4	5	
			3	× 6 — 1 = 17
	69	13	3	= 3 times.
			6	
	417	19	6	= 18 times.
	23	4	5	= once.
	£394	15	1	= 17 times.

Generally, therefore, find two numbers whose product nearly equals the multiplier, and find the *difference* between this product and the multiplier. Multiply the sum by one of these numbers, and the product by the other, and *add* or *subtract* as many times the first line (or original sum) as this difference denotes: *adding* if the product of the number is *less* than the multiplier, and *subtracting* if it is *larger*.

Exercise 67.

1. Multiply 4*l.* 4*s.* 10¼*d.* by 13.
2. Multiply 8*l.* 16*s.* 4*d.* by 158.
3. Multiply 1*s.* 10*d.* by 144.
4. Multiply 10*l.* 0*s.* 5¾*d.* by 108.
5. Add 14,147*l.*, 16*s.* 5*d.*; 8121*l.* 11*s.* 3¾*d.*; 2148*l.* 13*s.* 10½*d.*; 914*l.* 8*s.* 7*d.*; 4125*l.* 16*s.* 4¾*d.*; 13,412*l.* 9*s.* 11*d.*
6. What is the difference between 5 times 481*l.* 11*s.* 6*d.*, and 7 times 369*l.* 8*s.* 4*d.*?
7. What is the difference between 27 times 1028*l.* 8*s.* 11*d.*, and 36 times 943*l.* 18*s.* 9*d.*?
8. In noting down any number, why do you put a cipher where there is no significant figure?

Exercise 68.

1. Multiply 240*l.* 11*s.* by 17.
2. Multiply 8174*l.* 8*s.* 11½*d.* by 25.
3. Multiply 91,827*l.* 16*s.* 4*d.* by 39.
4. Multiply 61,427*l.* 18*s.* 11½*d.* by 54.
5. From 814,157*l.* 11*s.* 6*d.*, take 641,728*l.* 19*s.* 10½*d.*
6. From 8279*l.* 1*s.* 4¾*d.*, take 6010*l.* 12*s.* 9*d.*
7. From 28 cwt. 2 qrs. 11 lb., take 15 cwt. 3 qrs. 20 lb.
8. From 164 yrs. 8 mo. 16 d., take 127 yrs. 9 mo. 24 d.

COMPOUND DIVISION

Is the method of dividing any compound number into any number of equal parts, or of finding what compound number, multiplied by the divisor, will be equal to the dividend.

Set down the dividend, with the divisor to its left hand, as in Simple Division.

If the divisor be a single figure, begin at the left hand or highest denomination of the dividend, and set down the quotient below, as in Simple Division.

If there be any remainder, it must be multiplied by the number of units in the next lower denomination which make one unit in this denomination, and be carried forward.

The sum formed by adding what is carried to the next denomination must then be divided by the same divisor; the *quotient* must be set down, and the remainder (if there be any) must be multiplied, as before, by the number of units in the next lower denomination, which make one unit in that denomination, and be carried forward.

Every denomination must be divided in the same manner; and the whole result thus obtained will be the answer required.

Thus, if it be required to divide 121*l.* 5*s.* 7*d.* by 2.

$$\begin{array}{rrrr} & \pounds. & s. & d. \\ 2) & 121 & 5 & 7 \\ \hline & 60 & 12 & 9\frac{1}{2} \\ \hline\hline \end{array}$$

Here 2 is contained in 12 six times without a remainder; we therefore set down 6 for the first figure in the quotient, and have nothing to carry: proceeding to the next figure, 2 is not contained in 1; we therefore set down a cipher, and have 1 to carry; but one pound is equal to 20 shillings; we therefore carry 20 to the shillings' denomination, and have 25, instead of 5, to divide by 2. Now 2 is contained in 25 12 times with 1 over; we therefore set down 12 for the quotient in this denomination, and carry 1 shilling; but 1 shilling = 12 pence, so that we must carry 12 to the pence denomination, and adding this to 7, we have 12 and 7, or 19, to divide by 2. Now 2 is contained in 19 9 times with 1 over; we therefore set down 9 for the quotient, and carry 1 penny; but 1 penny equals 4 farthings: to this we have nothing to add, as there are no farthings in the dividend, we must therefore divide 4 by 2, this gives a quotient 2, and no remainder; we therefore have to set down 2 farthings, or 1 halfpenny: and the whole quotient is 60*l.* 12*s.* 9½*d.*

Let it be required to divide 73*l.* 7*s.* 3*d.* by 6.

$$\begin{array}{rrrr} & \pounds. & s. & d. \\ 6) & 73 & 7 & 3 \\ \hline & 12 & 4 & 6\frac{1}{2} \\ \hline\hline \end{array}$$

Here 6 is contained in 7 once, with a remainder 1: we therefore set down 1 for the quotient, and carry 10; because 1 in the place of tens is equal to 10 in the place of units; we have, therefore, next to divide 13 by 6: this gives a quotient 2, and 1 over. Carrying, therefore, 1 pound, or 20 shillings, we have 27 shillings to divide by 6, which gives a quotient 4, and 3 over: but 3 shillings equal 36 pence, we have therefore 36 to carry to the pence, which gives, with the 3 there, 39 to be divided by 6: this, divided by 6, gives a quotient 6, and 3 over; but 3 pence equal 12 farthings, we have, therefore, 12 to divide by 6, which gives a quotient 2, and nothing over. We have, therefore, 2 farthings, or 1 halfpenny, to set down.

And the whole result is 12*l*. 4*s*. 6½*d*.

Exercise 69.

1. Divide 143*l*. 5*s*. by 2.
2. Divide 741*l*. 16*s*. 8½*d*. by 2.
3. Divide 325*l*. 16*s*. 3*d*. by 4.
4. Divide 347*l*. 14*s*. 6*d*. by 7.
5. From 81,414*l*. 12*s*. 6*d*. take the half of 158,120*l*. 18*s*. 6*d*.
6. What is the difference between 4 times 807*l*. 4*s*. 6*d*., and the fourth part of 2804*l*. 16*s*. 8*d*.?
7. A and B entered into partnership: A brought a capital of 49,697*l*. 2*s*., which was 12 times as great as B's capital. What was their united capital?
8. Explain the method of carrying in Compound Division.

If the number to be divided consists of *weights* instead of money, the work must be done exactly in the same manner; only we must have recourse to the table of weights, instead of the table of money, to ascertain what to carry.

And whatever number is over in any denomination, must be multiplied by the number of units in the next lower denomination, which is contained in one unit of that denomination, before it be carried.

Thus, if it be required to divide 4 cwt. 3 qrs. 17 lb. 8 oz. by 2.

	cwts.	qrs.	lb.	oz.
2)	4	3	17	8
	2	1	22	12

Dividing the 4 hundredweight by 2, we have a quotient 2, and no remainder: dividing the 3 quarters by 2, we have a quotient 1, and 1 over; but one quarter equals 28 pounds; we have then 28 to carry to the pounds: this, with 17, makes 45 pounds to be divided, which, when divided by 2, gives a quotient 22, and 1 over. We have, therefore, 1 pound or 16 ounces, to carry, and this, added to 8 ounces, gives us 24 ounces to be divided, which, divided by 2, gives a quotient 12, without any remainder. And our whole result is 2 cwt. 1 qr. 22 lb. 12 oz.

Exercise 70.

1. Divide 323 cwt. 3 qrs. by 5.
2. Divide 1019 cwt. 0 qrs. 16 lb. by 8.
3. Divide 603 years 2 mo. 3 weeks by 5.
4. Divide 1157*l.* 3*s.* 6*d.* by 9.
5. Divide 5796*l.* 2*s.* 8*d.* by 8.
6. A received two packages: one weighed 8 ton 7 cwt. 1 qr. 27 lb.; and the other 9 ton 6 cwt. 3 qrs. 4 lb.: what was the whole weight?
7. A had 816*l.* 16*s.* 9*d.*; B had 390*l.* 17*s.* 9*d.* more than A: how much had they altogether?
8. If 41*l.* 2*s.* 9*d.* be divided between 9 persons, what will be the share of each?

When the divisor is above 12, and is the product of two simple figures, the result may be obtained by dividing *by one of these figures*, and then dividing the *quotient so obtained by the other*.

Thus, if 16 be the divisor, we may divide by 4 and 4, because 4 times 4 equal 16; or if 21 be the divisor, we may divide by 7 and 3, because 3 times 7 = 21; and so on.

To divide 2178*l.* 8*s.* 9*d.* by 15.

Here our divisors may be 3 and 5, because 3 times 5 = 15.

$$15=\left\{\begin{array}{l}\\ 3)\\ \\ 5)\\ \\ \end{array}\right.\begin{array}{rrr}\pounds. & s. & d.\\ 2178 & 8 & 9\\ \hline 726 & 2 & 11\\ \hline 145 & 4 & 7\\ \hline\hline\end{array}$$

Dividing first by 3, we obtain a quotient 726*l.* 2*s.* 11*d.*; and dividing this by 5, we obtain the quotient 145*l.* 4*s.* 7*d.*: the same sum which, when multiplied by 15[1], gave our dividend for the result.

It can make no difference in the result, whether we divide by the 3 or the 5 first.

$$15=\left\{\begin{array}{l}\\ 5)\\ \\ 3)\\ \\ \end{array}\right.\begin{array}{rrr}\pounds. & s. & d.\\ 2178 & 8 & 9\\ \hline 435 & 13 & 9\\ \hline 145 & 4 & 7\\ \hline\hline\end{array}$$

In this sum the answer is exactly the same as when we divided first by the 3, and afterwards by the 5.

Exercise 71.

1. Divide 437*l.* 18*s.* 6*d.* by 9.
2. Divide 564*l.* 16*s.* 6*d.* by 12.
3. Divide 876*l.* 13*s.* 8*d.* by 16.
4. Divide 357*l.* 11*s.* 6*d.* by 54.
5. Divide 137*l.* 4*s.* $7\frac{3}{4}$*d.* by 144.
6. Multiply 49*l.* 13*s.* 6*d.* by 7.
7. What will be the cost of 4 horses at 81*l.* 9*s.* $0\frac{3}{4}$*d.* each?
8. At 17 minutes past 10 o'clock, how long does it want to 25 minutes past 6 o'clock?
9. A had 48 times 613*l.* 8*s.* 6*d.*; and B had 56 times 476*l.* 9*s.* 11*d.*: what had they together? and how much had A more than B?
10. Divide 87,474*l.* by 72.

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[1] See page 68.

Let it be required to divide 1061 cwt. 2 qrs. by 28.

The 28 may be broken into 4 and 7, because 4 times 7 = 28.

$$28=\left\{\begin{array}{r}4)\\ \\ 7)\end{array}\right.\begin{array}{rrr}\text{cwt.}&\text{qrs.}&\text{lb.}\\ 1061&2&0\\ \hline 265&1&14\\ \hline 37&3&18\\ \hline\hline\end{array}$$

Here we begin by dividing 1061 by 4, which gives a quotient 265, and 1 over: 1 hundredweight equals 4 quarters; we have therefore 4 to carry to the quarters, which, with the 2 quarters in the original quantity, gives 6 quarters: 4 is contained in this *once*, with a remainder 2, we therefore set down 1, and carry 2 quarters; but 2 quarters equal 56 pounds; we therefore carry 56 to the pounds, and dividing this by 4, we obtain a quotient 14, and no remainder.

Then dividing this result by 7, we obtain a quotient 37 in the hundredweights, with a remainder 6: but 6 hundredweight = 24 quarters; we have therefore 24 to carry, and adding this to the 1, we have 25 for a dividend: 7 is contained in 25 3 times, with a remainder 4; we therefore set down 3, and carry 4 quarters; but 4 quarters = 112 pounds: we have therefore 112 pounds to carry, which, with 14 pounds, gives 126 for a dividend, in which 7 is contained exactly 18 times. So that our whole result is 37 cwt. 3 qrs. 18 lb.

Had there been any remainder in the last division, we must have carried it forward to the ounces' denomination, and have divided again, always continuing the work until there is no remainder, or no lower denomination.

Exercise 72.

1. Divide 134 cwt. by 16.
2. Divide 267 cwt. 3 qrs. by 28.
3. Divide 46 years 6 mo. by 56.
4. Divide 628 lb. 3 ℥ 7 ʒ by 49.
5. What is the difference between 4 times 81*l.* 9*s.* 0¾*d.*, *and* 7 *times* 49*l.* 13*s.* 6*d.*?

6. If 7 quarters of wheat cost 15*l.* 17*s.* 5¾*d.*, what is that per quarter?

7. Suppose a labourer earns 8*l.* 13*s.* 5¼*d.* in 9 weeks, how much is that per week?

8. Add the following sums:—1549*l.* 17*s.* 11½*d.*; 636*l.* 12*s.* 7*d.*; 954*l.* 14*s.* 6¼*d.*; 1235*l.* 7*s.* 4¾*d.*; 154*l.* 6*s.* 2½*d.*; 93*l.* 1*s.* 7¼*d.*

When the divisor is not the product of 2 simple numbers, we must divide by it at once, after the manner of Long Division, multiplying each remainder in its turn by the number of units in the next lower denomination, which make one unit in that denomination, and adding to it the units of this latter denomination in the original sum, and proceeding with it as a new dividend.

Thus, let it be required to divide 199*l.* 3*s.* 10*d.* by 53.

```
      £.   s.  d.   £.   s.  d.
  53)199   3   10  (3   15   2
     159
     ———
      40
       20
     ———
      803
      53
     ———
      273
      265
      ———
        8
       12
      ———
       106
       106
      ═══
```

We first see how often 53 is contained in 199, which gives us a quotient 3, and a remainder 40: we have then to multiply the remainder by 20, to bring it into the denomination shillings, which would give a product 800, but we add with this the 3 shillings in the original dividend, and have for a new dividend 803: this contains 53 15 times, with a remainder 8. This remainder is multiplied by 12, to bring it into the denomination pence; and, with the 10 in the original dividend added to it, gives 106 for a new dividend, in which 53 is contained exactly twice; so that, on the whole, 199*l.* 3*s.* 10*d.*, divided by 53, gives a result 3*l.* 15*s.* 2*d.*

Care must be taken in this method of division to distinguish between the different denominations in the quotient, marking off for *shillings* the figures of the quotient which were obtained after we multiplied by 20; for pence, those obtained after multiplying by 12; and for farthings, those obtained after multiplying by 4.

Exercise 73.

1. Divide 213*l.* 5*s.* by 60.
2. Divide 5275*l.* 17*s.* 5¼*d.* by 23.
3. Divide 87,342*l.* 15*s.* 7½*d.* by 128.
4. Divide 1407*l.* 17*s.* 7*d.* by 243.
5. If 12 pigs cost 11*l.* 17*s.* 6*d.*, what is the cost of one?
6. Multiply 3*s.* 7¼*d.* by 767.
7. Multiply 7*l.* 14*s.* 9½*d.* by 3645.
8. What is the difference between the ninth part of 3338*l.* 6*s.* 10½*d.*, and the half of 143*l.* 5*s.*?

Exercise 74.

1. Divide 3602 days 0 hour 30 min. by 94.
2. Divide 91,801 days 3 hours by 39.
3. Divide 14,010 hhd. 16 gal. 3 quarts of wine by 54.
4. Divide 50 yards 1 qr. by 12.
5. Divide 39,287 miles 6 fur. 32 poles by 17.
6. Divide 2219 lb. 8 ℥ 7 ʒ 0 ℈ 14 gr. by 11.
7. A school was to break up on the 20th of June; how long did it want to the time on the 27th of April?
8. If A walks daily 17 miles 4 furlongs, how far will he walk in a year?

If the number consists of *weights* or *periods of time*, instead of sums of money, the work must be done in the same way, using the tables of weights, or the table of time, for finding *the multipliers* of the remainders.

Divide 64 months 2 weeks 5 days 22 hours by 34.

$$
\begin{array}{rrrrrlrrr}
 & \text{mo.} & \text{wk.} & \text{ds.} & \text{hrs.} & \text{mo.} & \text{wk.} & \text{ds.} & \text{hrs.}\\
34) & 64 & 2 & 5 & 22 & (1 & 3 & 4 & 7\\
 & 34 \\
\cline{2-2}
 & 30 \\
 & 4 \\
\cline{2-2}
 & 122 \\
 & 102 \\
\cline{2-2}
 & 20 \\
 & 7 \\
\cline{2-2}
 & 145 \\
 & 136 \\
\cline{2-2}
 & 9 \\
 & 24 \\
\cline{2-2}
 & 238 \\
 & 238 \\
\cline{2-2}
 & \cdots \\
\cline{2-2}
\end{array}
$$

Here we have 34 contained in 64 *once*, with a remainder 30; multiplying this remainder by 4, to reduce it to weeks, and adding the 2, we have 122 for the next dividend: this contains 34 3 times, with a remainder 20: multiplying this remainder by 7, to reduce it to days, and adding the 5, we have 145 for the next dividend; in this 34 is contained 4 times, with a remainder 9; multiplying this remainder by 24, to reduce it to hours, and adding the 22, we have 238 for the next dividend, which contains 34 exactly 7 times. And the whole quotient is 1 month 3 weeks 4 days 7 hours.

Exercise 75.

1. Divide 142 cwt. 1 qr. 14 lb. by 17.
2. Divide 47 years 3 mo. 3 weeks 6 days by 57.
3. Divide 296 cwt. 1 qr. 21 lb. by 31.
4. Divide 653 lb. 11 ℥ 5 ʒ by 51.

5. A gentleman left by will 100*l.* 10*s.* 6*d.* each to threescore poor persons; how much did he thus leave to charitable purposes?

6. A postman walked 18 miles every day for 24 years; what distance did he walk in that time?

7. A gentleman spent in snuff 9 pence daily; how much money did he waste in a year?

8. A gentleman interested in missionary objects determined to practise great self-denial, in order to give more largely to that cause. He diminished his expenditure daily by 2*l.* 4*s.* 6*d.* How much was he thus enabled to give to the missionary society in 27 years?

FRACTIONAL MULTIPLIERS.

By the aid of Compound Division, we are enabled to multiply by such numbers as *six and a half* (6½): *twenty-seven and a quarter*(27¼): *nine and three quarters* (9¾), &c.

In such cases we multiply by the integral part of the multiplier, as in Compound Multiplication, and add to the product that part of the original sum which is represented by the fractional part of the multiplier.

Thus, to multiply 214*l.* 6*s.* 4*d.* by 6½, we must multiply the sum by 6, and add to the product half the original sum.

£.	*s.*	*d.*
214	6	4
		6½
1285	18	0
107	3	2
£1393	1	2

To multiply 118*l.* 3*s.* 4*d.* by 12¼.

£.	*s.*	*d.*
118	3	4
		12¼
1418	0	0
29	10	10
£1447	10	10

Here we multiply the sum by 12, then place under the product the quotient arising from the division of the same sum by 4; and adding these sums together, we have a result equal to *twelve and a quarter* times the original sum.

To multiply 27*l.* 8*s.* 7*d.* by 9¾.

£.	*s.*	*d.*
27	8	7
		9¾
246	17	3
20	11	5¼
£267	8	8¼

£.	*s.*	*d.*
27	8	7
		3
4)82	5	9
£20	11	5¼

Here we multiply the original sum by 9, which gives a result 246*l.* 17*s.* 3*d.*: then, to find the value of *three quarters* of the original sum, we multiply it by 3, and divide the product by 4. This gives 20*l.* 11*s.* 5¼*d.*; which, when added to the original product, gives 267*l.* 8*s.* 8¼*d.*, as the product of 27*l.* 8*s.* 7*d.*, multiplied by *nine and three quarters.*

Exercise 76.

1. Multiply 11*s.* 7¼*d.* by 9½.
2. Multiply 1*l.* 7*s.* 8*d.* by 85¾.
3. Multiply 4*l.* 6*s.* 3*d.* by 117¼.
4. Multiply 5*l.* 4*s.* 3½*d.* by 215½.
5. Multiply 3*l.* 15*s.* 5¼*d.* by 874¾.
6. If I have an allowance of 1*l.* 17*s.* 11*d.* weekly, what may I spend daily?
7. How would you find the fractional part to be added, if the multiplier were 27⅘, or 36⅜?
8. A merchant received 31 packages, each weighing 9 cwt. 2 qr. 7 lb.: what was the united weight of the whole?

REDUCTION

Is the method of bringing numbers from one denomination into another, without altering their value.

When any simple quantity is to be reduced from a higher denomination to a lower, we must multiply it by the number of units in the lower denomination, which make one unit in the higher denomination: that is to say,

To reduce pounds to shillings, we must multiply by 20.

To reduce shillings to pence, we must multiply by 12.

To reduce pence to farthings, we must multiply by 4.

If we have to reduce pounds to pence, we must multiply first by 20 to reduce it to shillings, and then by 12 to reduce it to pence.

If we have to reduce pounds to farthings, we must multiply by 20 to reduce it to shillings; then by 12, to reduce it to pence; and, lastly, by 4, to reduce it to farthings.

Thus, to reduce 5*l.* to farthings:—

$$\begin{array}{rl} £5 & \\ 20 & \\ \hline 100 & \text{shillings.} \\ 12 & \\ \hline 1200 & \text{pence.} \\ 4 & \\ \hline 4800 & \text{farthings.} \\ \hline\hline \end{array}$$

Exercise 77.

1. In 7*l.*, how many pence are there?
2. In 10*l.*, how many shillings are there?
3. In 9*l.*, how many farthings are there?
4. In 29*l.*, how many pence?
5. In 560*l.*, how many shillings?
6. In 9874*l.*, how many pence?
7. In 8147*l.*, how many farthings?
8. In 216*l.*, how many halfpence?
9. In 15*s.*, how many farthings?

If the number to be reduced represents a weight, or a portion of time, the proper table of weights, or of time, must be used to find the different multipliers.

Thus, to reduce 14 months to days, we must first multiply the 14 by 4, to reduce it to weeks, and then we must multiply the result by 7, to reduce it to days.

Thus, 14 months.
4
56 weeks.
7
392 days.

And we find that 14 months contain or equal 392 days.

Or, if we have to reduce 27 cwt. to pounds, we must multiply first by 4, to reduce it to quarters, and then we must multiply the result by 28, to reduce it to pounds.

Thus, 27 cwt.
4
108 qrs.
28
864
216
3024 lbs.

And we find that 27 cwt. = 3024 pounds.

Exercise 78.

1. In 129 cwt., how many quarters?
2. In 63 cwt., how many pounds?
3. In 9 years, how many weeks?
4. In 7 years, how many days?
5. In 27 quarters of corn, how many pecks are there?
6. In 87*l*., how many farthings?
7. In 1260*l*., how many pence?
8. A was in debt to B 12 shillings, and gave him 1 pound, asking for the necessary change, which B returned in *halfpence:* how many did he give him?

When the quantity to be reduced is a compound number, multiply the number in the highest denomination by the number of units in the next lower denomination, which make one unit in that denomination, and to the product add the number which was in this lower denomination before.

The number so found must then be multiplied by the number of units in the *next* lower denomination, which make one unit in it, and to the product must be added the number, which was in this lower denomination before.

Proceed in the same way through every denomination, and the last result will be the value, in the lowest denomination, of all the numbers in the higher denominations together.

How many farthings are equal in value to 126*l.* 14*s.* $6\frac{1}{2}$*d.*?

	£.	s.	d.		£.	s.	d.
1.	126	14	$6\frac{1}{2}$	2.	126	14	$6\frac{1}{2}$
	20				20		
	2520				2534		
	14				12		
	2534 shillings.				30414		
	12				4		
	30408				121658		
	6						
	30414 pence.						
	4						
	121656						
	2						
	121658 farthings.						

Here we multiply 126 by 20, to reduce it to shillings, which gives a product 2520; but we have to add to this the 14 shillings already in this denomination, and have therefore a result, 2534 shillings: multiplying this by 12 *to re*duce it to pence, and adding in the 6 already in this denomination, we have a result 30,414 pence; and multiplying this by 4, to reduce it to farthings, and adding in two farthings already in this denomination, we have a result 121,658 farthings.

In (1) the integers of the different denominations are placed *under* the respective products, and added to them afterwards. In (2) these integers are added at the time of multiplying, and this is the method usually followed.

Exercise 79.

1. In 127*l*. 3*s*., how many pence?
2. In 196*l*. 18*s*., how many shillings?
3. In 12*s*. 6*d*., how many farthings?
4. In 168 cwt. 3 qrs., how many quarters?
5. In 3 years, how many days?
6. In 54 ounces of silver, how many grains?
7. A received in shillings a payment equal to 23*l*. 14*s*.; how many did he receive?
8. What is the difference between 1*l*. and 240 pence?

When any simple quantity is to be brought from a *lower* denomination to a higher, we must proceed in a manner just the reverse of the last rule; that is to say, we must *divide* by the number of units in the *lower* denomination, which make *one unit* in the higher denomination, instead of multiplying by that number.

Thus, to bring *shillings* into *pounds*, we must divide by 20; and, to bring *pence* into *shillings*, we must divide by 12.

To bring *pence* into *pounds*, we must first divide by 12 to bring them into shillings, and then divide the quotient so found by 20, to bring it to pounds.

Thus, if it be required to reduce 1200 pence to pounds, or to find how many pounds are equal to 1200 pence,

12) 1200 pence.

2,0) 10,0 shillings.

5 pounds.

We first divide 1200 by 12 to reduce it to shillings; and find that 1200 pence = 100 shillings: we then divide 100 by 20, to reduce it to pounds, and find that it equals 5*l.*

If we have to reduce farthings to pounds, we must divide first by 4, to reduce them to pence; then divide that quotient by 12, to reduce it to shillings, and then divide this second quotient by 20, to reduce it to pounds.

Thus, if it be required to find how many pounds are equal to 4800 farthings,

```
   4 ) 4800 farthings.
       ----
  12 ) 1200 pence.
       ----
 2,0 ) 10,0 shillings.
       ----
          5 pounds.
       ====
```

And thus we find that 4800 farthings = 1200 pence = 100 shillings = 5 pounds.

Exercise 80.

1. How many pounds are there in 480 pence?
2. How many pounds are there in 2880 farthings?
3. Reduce 6960 pence to pounds.
4. If I owe a debt of 2,369,760 pence, how many pounds will discharge it?
5. A owes B 220*l.*, and pays him 103,680 half pence, how much remains due?
6. How many farthings are there in 45*l.* 17*s.* 6¼*d.*?
7. How many farthings are there in 99*l.* 19*s.* 11¾*d.*?
8. What is the difference between 23,999 pence, and 4 times 24*l.* 19*s.* 11*d.*

In the same way we can reduce minutes, hours, days, &c., to a higher denomination by using the proper divisors found in the Table of Time; or we can reduce weights, such as *drams, ounces,* pounds, &c., to a higher denomination by using the proper divisors found in the respective Tables of Weights.

Thus to find how many months are equivalent to 392 days,

$$\begin{array}{r l} 7\,)\,\underline{392} & \text{days.} \\ 4\,)\,\underline{56} & \text{weeks.} \\ 14 & \text{months.} \end{array}$$

Here we first divide the days by 7, to reduce them to weeks, and then we divide the quotient 56 by 4, to reduce it to months; and we find that 392 days equal 56 weeks, equal 14 months.

Or, to find how many hundredweight are equal to 3024 pounds.

$$28\begin{cases} 4\,)\,\underline{3024} \text{ lbs.} \\ 7\,)\,\underline{756} \end{cases}$$

$$\begin{array}{r l} 4\,)\,\underline{108} & \text{qrs.} \\ 27 & \text{cwt.} \end{array}$$

We, in this case, divide first by 28 (or by 4 and by 7) to reduce the pounds to quarters; and then we divide the quarters by 4, to reduce them to hundredweights; and we find that 3024 pounds equal 108 quarters, or equal 27 hundredweight.

Exercise 81.

1. In 5 lb. 2 oz. 14 dwts. of silver, how many pennyweights?
2. In 21 ton 15 cwt., how many cwts.?
3. In 17 cwt. 3 qrs. 15 lb., how many pounds?
4. Reduce 420 cwt. to tons.
5. In 30 guineas how many shillings are there, and how many pence?
6. What is the difference between 2003 lbs., and 32,048 ounces?

7. How many nails are there in 4 yards 3 qrs. 3 nl.?

8. How many grains are equal to 27 lbs. 7℥ 2ʒ 1℈ 2 grs?

Sometimes the divisor is not contained an exact number of times in the dividend: in that case the remainder must be set down to represent the units in the denomination, from which we are then reducing the number.

Thus, if we are reducing *shillings* to *pounds*, the remainder will represent the *shillings* which are over: if we are reducing *pence* to *shillings*, the remainder will represent the *pence* which are over.

Thus, to find the value of 28,141 farthings, in pounds, shillings, and pence,

$$\begin{array}{r|r r}
4) & 28141 & \\ \cline{2-3}
12) & 7035 & \frac{1}{4} \\ \cline{2-3}
2,0) & 58,6 & 3\frac{1}{4} \\ \cline{2-3}
 & \pounds 29\ \ 6 & 3\frac{1}{4} \\ \cline{2-3} \cline{2-3}
\end{array}$$

Here, dividing by 4, we have a quotient 7035, and a remainder 1; and, as the sum we are dividing is farthings, this remainder is 1 farthing; we therefore set it down as such, with a mark of separation between it and the quotient. We then divide the quotient by 12, to bring it into shillings, and obtain a quotient 586, with a remainder 3, or 3 pence over; we therefore set down this remainder in a similar manner for the pence. We then divide the quotient by 20, to reduce it to pounds, and obtain a quotient 29, with a remainder 6, or six shillings over; which we set down to represent the shillings. And bringing down the successive remainders, and arranging them in order, we have obtained the result, 29*l.* 6*s.* 3¼*d.* as being equal to 28,141 farthings.

If the numbers represent weights, or spaces of time, the remainders must be set down in a similar manner; and they are always in the denomination *from* which we are reducing, *that is,* in the denomination of the *dividend.*

Exercise 82.

1. Reduce 5688 pence to shillings and pounds.

2. How many pounds, shillings, &c., are equal to 95,996 farthings?

3. Reduce 159,020 grains of medicine to pounds, ounces, &c.

4. When A had walked 84,480 feet, how many miles had he gone?

5. How many leagues are equal to 190,080 yards?

6. How many Flemish ells must I receive in return for 198 English ells?

7. In 26 Flemish ells 2 quarters, how many nails are there?

8. From 24,146*l.* 11*s.* 7*d.*, take 19,816*l.* 10*s.* 9*d.*

9. A poor man who was interested in the cause of Christ, worked one hour daily to earn something for the Missionary Society. His earnings were at the rate of 2 shillings in 5 hours: what did he contribute in a year?

When the divisor is greater than 12, and is the product of two numbers, the proper result may be obtained by dividing it *first by one of those numbers*, and then dividing the *quotient* so found by the *other*.

Thus, to find how many quarters there are in 252 pounds.

28 lbs. = 1 quarter; we have therefore to divide by 28: but 28 = 4 × 7, we may therefore divide first by 4, and then by 7,

$$28\left\{\begin{array}{r} 4)\underline{252} \\ 7)\ \underline{\ 63} \end{array}\right.$$
$$\underline{\underline{9}}$$

and we find that 252 pounds = 9 quarters.

The divisors may always be thus broken when it is possible, and the work is shortened, and rendered more easy by it.

Exercise 83.

1. In 896 pounds, how many hundredweights are there?

2. In 3024 pounds, how many hundredweights are there?

3. In 7680 poles, how many miles are there?

4. In 3492 bushels of coal, how many chaldrons are there?

5. In 6048 quarts of wine, how many hogsheads are there?

6. In 9072 pints of wine, how many tierce are there?

7. If 88*l.* 4*s.* 0*d.* were divided between 56 widows, how much would each have?

8. If 2068 persons in distress had the sum of 977*l.* 19*s.* 10*d.* distributed amongst them, what would be the share of each?

9. In 324 moidores[1], how many pounds are there?

But when the divisors are not exactly contained in the dividends, there is need of great care to set down the remainders, so as accurately to represent the proper number in the denomination from which we are reducing.

Thus to find the value of 63 lb. in quarters and pounds, we must divide by 28, or 4 and 7, because 4 times 7 = 28.

$$28\left\{\begin{array}{rrr} 4) & 63 & \\ \cline{2-3} 7) & 15 & 3 \end{array}\right.$$
$$\begin{array}{rr} \hline 2 & 7 \\ \hline\hline \end{array}$$

Here we find that 63 contains 4 15 times, with a remainder 3, or with 3 lb. over. Then dividing the quotient 15 by 7, we obtain a second quotient 2, and we have a remainder 1: we must, however, remember that this is not 1 *pound*, but one *four-pound*, because the dividend in this case is *pounds divided by* 4: we must therefore add (not 1, but) 4 to the 3 lbs. in the line above, to represent the true number of pounds remaining, and we have therefore 4 + 3 or 7 pounds remaining. And thus 63 lbs. = 2 qrs. 7 lbs.

---

[1] A moidore = 27 shillings.

Again, let it be required to find the value of 68 pounds in quarters and pounds, &c.

$$\begin{array}{l} 28=\left\{\begin{array}{l} 4)\,68 \\ \quad\overline{} \\ 7)\,17 \end{array}\right. \\ \hline \qquad\quad 2 \quad 12 \\ \hline\hline \end{array}$$

Here we have no remainder, when we divide by 4 : but when we divide by the 7, we have a remainder 3 ; this is not, however, 3 *pounds*, but 3 *four-pounds*, because the pounds have already been divided by 4 : so that our true remainder is three times 4, or 12 pounds. And thus 68 lbs. =2 qrs. 12 lbs.

Let it therefore be observed, that in dividing by two numbers, whose product is equal to the divisor, the remainder (if there is any) arising from the division by the *first* number must be set down *as it is;* and when we divide by the *second* number, the remainder (if there is any) must be *multiplied by the first divisor* before it is set down, and the *two remainders added together* will represent the true number in that denomination.

Find the value of 14,145 ounces in quarters, pounds, &c.

$$\begin{array}{l} 16=\left\{\begin{array}{l} 4)\,14{,}145 \\ \quad\overline{} \\ 4)\;\;3536 \quad 1 \end{array}\right. \\ \hline 28=\left\{\begin{array}{l} 4)\;\;884 \\ \quad\overline{} \\ 7)\;\;221 \end{array}\right. \\ \hline \qquad\quad 31 \quad 16 \quad 1 \\ \hline\hline \end{array}$$

Here we use 4 and 4 as divisors instead of 16: and 4 and 7 instead of 28.

And dividing by the first 4, we have a remainder 1 ; but when we divide by the second 4, we have no remainder; therefore 1 will represent the whole number of ounces remaining. Then proceeding to the reduction into quarters, the division by 4 leaves no remainder, and the division by 7 leaves a remainder 4 : but this is not 4 *pounds*, but 4 *four-pounds,* because the pounds have already been divided by 4, or brought to a denomination 4 times as great; so that in reality the remainder is *four times four pounds,* or 16 pounds. And the value of 14,145 ounces is 31 quarters, 16 pounds, 1 ounce.

Exercise 84.

1. In 1134 gallons of beer, how many hogsheads are there?
2. How many bushels are there in 1940 pints?
3. How many hours are there in a Julian year?
4. How many days are there in 10,946,880 minutes?
5. Reduce 279,872 pecks of wheat to quarters.
6. In 21,568 gallons of beer, how many half barrels are there?
7. How many days are there between the 21st of March, and the 16th of November?
8. If Henry has 517*l.* 6*s.* 8*d.*; and John has three times as much as Henry; and William has twice as much as John: how much have they altogether?
9. If Henry has 312*l.* 8*s.* 0*d.*; and James has 4*l.* 9*s.* 6*d.* less than twice this sum; and John has 27*l.* 8*s.* 4*d.* more than half the same sum: how much have they in all?

Exercise 85.

1. Reduce 435 hundredweights to tons.
2. Reduce 32,048 ounces to hundredweights.
3. What is the value in yards, &c., of 79 nails?
4. How many yards are there in 428 nails?
5. Reduce 320 nails to Flemish ells.
6. Having travelled 483 miles, 7 furlongs, 32 poles, how much remains to complete 500 miles?
7. A gentleman on joining the Temperance Society, devoted his savings to a charity. His wine formerly cost him 7*s.* 6*d.* daily: how much had he at his disposal every year?
8. A poor man unable to find employment at home, went every morning to a village 3 miles off, where he obtained work. How many miles were thus added to his labour in 43 years?

When the divisors are greater than 12, and are not *exactly the* product of two numbers, we must proceed according to the rules for long division[1]; setting down each remainder for the number in that denomination, and reducing each quotient in turn to the next higher denomination.

Exercise 86.

1. How many miles are there in 84,480 feet?
2. Reduce 58,734,720 inches to miles.
3. Which is the greater quantity—927 miles, or 4,894,560 feet?
4. Reduce 6,248,620 yards to miles.
5. In 1,136,112 poles, how many miles are there?
6. Divide 1,631,520 by 1760.
7. How many times would the wheel of a coach, which is 21 feet in circumference, turn round in going from Stamford to London, the distance being 89 miles?
8. How many barley-corns will reach round the earth, the circumference being considered 25,020 miles?

Exercise 87.

1. How many pints of wine are equal to 11,318,033 gallons?
2. In 70 pipes of port wine, how many gallons are there?
3. Having in my cellar 70,560 pints of wine, how many pipes have I?
4. In 74,963 quarts of sherry, how many pipes are there?
5. How many hogsheads are there in 54 barrels of ale?
6. How many quarters of wheat are equal to 279,872 pecks?
7. Reduce 525,960 minutes to years.
8. A drunkard was in the habit of drinking 3 quarts 1 pint of beer daily: how much did he drink in 38 years?
9. A merchant delivered to his correspondent as much cloth as amounted in value to 547*l.* 14*s.* 7*d.*, and received his money in dollars, worth 4*s.* 2*d.* each: how many did he receive?

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[1] See pages 35 and 36.

THE RULE OF THREE

Is the method of finding a number which shall bear the same proportion to a given number, as two other numbers bear to each other.

In the Rule of Three, as its name denotes, three terms are always given, and a fourth has to be found.

The three given numbers must be set down in one line, according to the following rule:

Observe which of the three given terms is of the same kind as the number to be found, and place it *last* in the line to the right hand.

Then consider whether from the nature of the question the answer will be *greater* or *less* than this number. If it will be *greater*, set down the greater of the other two numbers in the middle, and the smaller to the left hand. But if the answer is required to be *smaller*, set down the smaller of the other two numbers in the middle, and the greater to the left hand.

Then reduce the two first numbers to the same denomination, and reduce the third number to the lowest denomination which is mentioned in it.

Then multiply together the second and third terms, and divide their product by the first. The quotient will be the number which it was required to find, and will be in the same denomination as that to which the third term has been reduced.

It will therefore be necessary to bring the quotient to the highest denomination it will admit of, in order to set down the answer in its proper form.

Thus, let it be required to find an answer to the following sum:

If 10 men can do a piece of work in 12 days, how many men can do the same work in 2 days?

The question requires us to find a number of men; therefore the ten will be the third, or right hand term. The work is also to be done by them in a shorter time: it is evident therefore that a *greater number* will be required. We must, then, put the *greater* of the other two numbers in the middle, and the smaller first.

And our statement will be,

As 2 days are to 12 days, so are 10 men to the number required, or simply,

$$\begin{array}{r} 2 \;:\; 12 \;::\; 10 \\ 12 \\ \hline 2)\,120 \\ \hline 60 \\ \hline\hline \end{array}$$

And multiplying the two last terms together, and dividing the product by the first, we find that 60 men will be required.

Again:

If 60 men can do a piece of work in two days, how many men will be required to do it in 12 days?

Here the answer must be men; therefore 60 will be our third term: but a longer time is allowed, so that *fewer* men will be required to do it, the *smaller* number must therefore stand in the middle, and the sum must be stated thus:

$$\begin{array}{r} 12 \;:\; 2 \;::\; 60 \\ 2 \\ \hline 12)\,120 \\ \hline 10 \\ \hline\hline \end{array}$$

And multiplying together the two last terms, and dividing their product by the first term, we find that 10 men only would be required.

Exercise 88.

1. If 3 yards of cambric cost 7 shillings, what is the cost of 24 yards?
2. If 12 men can do a piece of work in 20 days, how many can do it in 8 days?
3. If 30 men can complete a work in 8 days, how many days will it take 16 men to do it?
4. If a man can walk 9 miles in 2 hours, how many will he walk in 24 hours?
5. How many silver fourpences will be equivalent to 432 moidores?

6. A person discharged a debt of 335*l.* 8*s.* 4*d.* with an *equal number* of each of the following coins, viz.: shillings, sixpences, fourpences, threepences, twopences, pennies, halfpennies, and farthings. What number of each did he give?

Let it be required to find an answer to the following sum. If 13 sheep cost 21*l.* 2*s.* 6*d.*, what will be the cost of 29 sheep?

Here the answer is to be money, so that 21*l.* 2*s.* 6*d.* must be the third term. It is also evident that the cost of 29 sheep will be greater than the cost of 13 sheep; so the *greater* of the other two numbers must be in the middle. And the question must be stated thus,

```
                       £.  s. d.
As,       13 : 29 ::   21  2  6
                        20
                       ---
                       422
                        12
                      ----
                      5070
                        29
                    ------
                     45630
                    10140
                    ------
                 13)147030(11310
                    13
                    --
                     17
                     13
                     --
                      40
                      39
                      --
                       13
                       13
                       --
                      ..0
```

We have here a sum of money consisting of pounds, shillings, and pence, as the price of 13 sheep; we must therefore reduce it to the lowest denomination mentioned in this compound number, that is, to *pence*, before we can conveniently multiply and divide by the two other terms. Having done this, we multiply, as in the last case, by the second term, 29, and divide the product by the first term, 13. This gives us a quotient 11310, which is the correct answer; but it is in the denomination *pence*, so that we must bring it into pounds, shillings, &c., by the rule for Reduction.

```
12) 11310
    ---------
20)  94,2 6
    ---------
     £47 2 6
    =========
```

And we find that the cost of 29 such sheep would be 47*l.* 2*s.* 6*d.*

Exercise 89.

1. If 8 pencil-cases cost 11 shillings, what will 96 pencil-cases cost?
2. If 48 yards of cloth cost 5*l.* 12*s.* 0*d.*, what is the cost of 3 yards?
3. If 8 sheep cost 17*l.* 12*s.* 0*d.*, what will 100 sheep cost?
4. If I can purchase 7 horses for 178*l.* 10*s.* 0*d.*, what will be the cost of 46 horses?
5. If 6 sheep cost 13*l.* 4*s.* 0*d.*, how many can I buy for 1100*l.*?
6. If I purchase 46 horses for 1173*l.*, how much shall I give for 40 horses?
7. How many coins worth sevenpence halfpenny will be equivalent to 1000*l.*?
8. From 16 lb. 9 ℥ 7 ʒ, take 14 lb. 11 ℥ 3 ʒ 2 ℈ 19 grs.
9. From 147 lb. 4 ℥ 1 ʒ, take 109 lb. 8 ℥ 7 ʒ 2 ℈.

Exercise 90.

1. What is the value of 3 chaldrons of coals at 1*s.* 6½*d.* a bushel?

2. If 3 lb. of tobacco cost 2*s.* 9*d.*, what will 255 lb. cost?

3. If 2 gallons of ale cost 1*s.* 9*d.*, what will one barrel cost?

4. If 32 gallons of brandy cost 49*l.* 12*s.* 0*d.*, what will 2 gallons cost?

5. If I can buy 3 yards of linen for 7 shillings, how much can I buy for 5*l.* 12*s.* 0*d.*?

6. If 350 men can complete a railroad in 9 months, in what time will ten men complete it?

7. How many pounds of gold are equal to 1254 pennyweights?

8. In 26 English ells, how many nails are there?

Exercise 91.

1. What is the value of 3 flocks of sheep, each containing 87, at the rate of 5*l.* for 4 sheep?

2. If 3 horses are worth 182*l.* 5*s.* 0*d.*, what is the cost of 24 such horses?

3. If 12 boys can eat 8 lbs. of cherries, how many pounds would 29 boys eat?

4. If I can walk 256 miles in 8 days, how far can I walk in 25 days?

5. A person undertook to walk 440 miles in 8 days, how far did he walk in a day? and what distance could he have completed in a month?

6. If the cost of maintaining 9 persons for one week is 3*l.* 7*s.* 6*d.*, what will be the cost of maintaining 250 persons for the same time?

7. Multiply 682*l.* 7*s.* 3¼*d.* by 128.

When the numbers which are given represent weights, measures, or time, each number must be reduced to the lowest denomination which is contained in it, by the proper table of weights, measures, or time, and the two first terms must be reduced to the *same denomination.*

Thus let the question be,

How many yards of matting, which is 2 feet 6 inches broad, will cover a floor which is 28 feet long and 24 feet broad ?

In this question the length and breadth of the room are given, and the breadth of the matting, we therefore have to find the *length of the matting;* our last term, then, will be *length*, namely 28. And as it is evident that *more* matting will be required for a width of 24 feet, than for a width of 2 feet 6 inches, our middle term will be the greater number of the other two.

The statement, then, will be this,

```
        ft. in.     ft.       ft.
As       2   6  :   24   ::   28
        12          12
        ——         ————
        30         288
        ══          28
                   ————
                  2304
                  576
                  ————
             3,0)806,4
                 —————————
                  268   24 feet
                        12
                       ————
               3,0)    28,8
                       ——————
                          9   18 inches
                               3
                              ——
                         3,0)5,4
                             ——————
                              1   24 barley corn.
                                  ══════
          3)268   9   1
            ——————————————
             89   1   9   1
            ══════════════
```

Here we reduce 2 feet 6 inches to inches, and also reduce 24 feet to inches, that the two first terms may be in the same denomination; we then multiply the second and third terms together, and divide by the first: this division gives a quotient 268 with a remainder 24, which is in the denomination feet, the same as the third term of the statement. We therefore multiply the remainder 24 by 12 to bring it into inches, and divide the product by 30; this gives a quotient 9 inches, and a remainder 18. This remainder we multiply by 3 to reduce it to barley corns, and again dividing by 30, we have a quotient 1 barley corn, and a remainder 24. So that our whole quotient is, 268 feet, 9 inches, 1 barley corn, and a remainder $\frac{24}{30}$.

And setting this down without the remainder, and dividing 268 by 3 to reduce it to yards, which gives a quotient 89 and 1 foot over, we find that the quantity of matting required is, 89 yards, 1 foot, 9 inches, 1 barley corn.

The division here has been performed by cutting off the cipher. The result would have been exactly the same if we had worked the sum by long division, thus,

```
           ft. in.      ft.        ft.
As          2   6   :   24    ::   28
           12           12
           --          ----
           30          288
           ==           28
                       ----
                       2304
                       576
                       ----
                   30)8064(268
                       60
                       ----
                       206
                       180
                       ----
                        264          18
                        240           3
 ft.   in.  b.         ----          --
3)268   9   1            24      30)54(1
 ------------            12          30
  89   1   9   1        ----          --
 ==============      30)288(9        24
                        270          ==
                        ----
```

It will at once be seen, that the remainders in this case are the same as in the former way of working the sum; and that the answers are the same in all respects.

Suppose it be required to find,

What is the value of 2 cwt. 3 qrs. of tea, at 5*s.* 6*d.* a pound?

Here the answer required is money, so that 5*s.* 6*d*, must be our third term: and the cost of 2 cwt. 3 qrs. must be greater than the cost of 1 lb. Our middle term must therefore be greater than the first.

The statement then will be this,

```
        lb.     cwt. qrs.      s.  d.
As      1   :    2    3   ::   5   6
                 4            12
                ---           ---
                11            66
                28            ===
               ----
               308
                66
               ----
              1848
             1848
            ------
        12)20328
           ------
        2,0)  169,4
             --------
               84  14
             ========
```

Here we reduce the second term to pounds, to make it of the same denomination as the first. Then, reducing the third term to pence, and multiplying the second and third terms together, we obtain a product 20,328. As the first term of our statement is 1, there can be no need of division; and 20,328 is the amount required in *pence* (the denomination to which the third term has been reduced). Then reducing these pence to shillings and pounds, we obtain the answer, 84*l.* 14*s.* 0*d.* as the price of 2 cwt. 3 qrs. of tea.

Exercise 92.

1. If a silver cup, weighing 3 lb. 11 oz., cost 8*l*. 18*s*. 2½*d*., what is the price per ounce?

2. If 2 cwt. 3 qrs. 21 lb. of coffee cost 27*l*. 8*s*. 4*d*., what is the price per ounce?

3. A gentleman has an income of 2000*l*. a year: his daily expenditure averages 5*l*. 15*s*.: does he live within, or does he exceed his income? and by what sum?

4. A labourer having thrashed 15 loads 1 qr. of wheat, received for his work 11*l*. 8*s*.: what was the price per quarter?

5. What will be the amount of assessment to the poor rate on 570*l*. 10*s*. 6*d*., at 3*s*. 6*d*. in the pound?

6. If 32 bricks will pave a space which is 1 yard square, how many bricks will be required to pave a floor 24 feet long and 18 feet wide?

7. If 3 lbs. of tea cost 16*s*. 4*d*., what will be the cost of a chest weighing 3 cwt. 2 qrs. 18 lbs?

8. How many perches are there in 55 acres, 2 roods, 38 perches?

Exercise 93.

1. If a family, consisting of 12 persons, consume 19 loaves of bread in a week, how many loaves will be consumed by an army of 27,000 men at the same rate?

2. If the cost of maintaining 4 paupers in the workhouse is 13*s*. 6*d*. weekly, what will be the cost of the whole establishment, consisting of 327 persons?

3. If 300 acres of land let for 750*l*. 12*s*. 8*d*., what is the rent per acre?

4. If the expense of cultivating 300 acres is 1586*l*. 10*s*., what will be the cost of cultivating 529 acres?

5. If a pedestrian can walk 9 miles in 2 hours 10 minutes, how far will he walk in 24 hours 30 minutes?

6. If a railway train travel at the rate of 85 miles in 2 hours, 7 minutes, 30 seconds, what distance will it complete in 3 days and nights?

7. How many barrels are equal to 6,284,565½ hogsheads of beer?

8. How many hogsheads of beer are there in 1134 gallons?

Sometimes all the numbers in the statement will require to be reduced to lower denominations.

If 2 cwt. 3 qrs. of tea cost 84*l.* 14*s.*, how much tea can be bought for 5*s.* 6*d.*?

Here the answer is to be "*weight*;" so that 2 cwt. 3 qrs. must be our third term. And it is evident that the quantity will be much smaller at a cost of 5*s.* 6*d.* than at a cost of 84*l.* 14*s.* The ***middle term*** must therefore be smaller than the first term.

The statement, then, will be,

```
         £.   s.       s.  d.        cwt. qrs.
As       84   14   :    5   6   ::     2   3
          20           12              4
        ------         --            ---
        1694           66             11
          12           ==             28
        ------                       ----
        20328                         308
        =====                          66
                                     ----
                                     1848
                                    1848
                                   ------
                          20328)20328(1
                                20328
                                -------
                                 .....
                                =======
```

Reducing the two first terms to pence, and the third term to *pounds*, and multiplying together the second and third terms and dividing by the first term, we have a quotient 1; and as the third term was reduced to the denomination pounds, this quotient is 1 lb.

The third term was reduced to *pounds*, because otherwise the dividend would have been less than the divisor.

Thus we find that 5*s.* 6*d.* would purchase 1 lb. of tea.

Exercise 94.

1. If the carriage of 2 cwt. 3 qrs. of goods cost 1*l.* 18*s.* 6*d.*, what will be the carriage of 20 ton 13 cwt. the same distance?

2. If I can travel 56 miles 3 furlongs for 1*l.* 2*s.* 6*d.*, what will be the cost of travelling 508¼ miles?

3. If 1 hogshead 8 gallons of wine cost 37*l.* 10*s.*, what will be the cost of 34 hhds. 5 gals.?

4. If 27 cows can be kept in a field 6 weeks, how long can 13 cows feed on the same pasture?

5. If 12 horses cost 1*l.* 3*s.* a day in hay and corn, for what sum can 57 horses be kept a year?

6. How much oil-cloth, 2ft. 6 in. wide, will cover a floor 38 feet long and 29 feet wide?

7. A had 21,414*l.* 11*s.* 6*d.*; B had half as much; C had one third of the sum; and D had as much as B and C together: how much had they in all?

8. What is the difference between 6 times 249*l.* 11*s.* 6*d.*, and 35 times 39*l.* 2*s.* 8*d.*?

THE DOUBLE RULE OF THREE

Is the method of finding a quantity by one operation, which according to the Simple Rule of Three would require two or more statements, to be worked out separately.

It enables us to find an answer to such questions as this:

If 8 men can dig 240 yards of earth in 6 days, how many men must there be to dig 200 yards in 4 days?

Here the answer is to be a number of men: therefore 8 will be the third term, and will be common to both statements; the first and second terms of one statement will be periods of time; and the first and second terms of the other statement will be yards of earth.

In making the two statements, after setting down the third term, we must ascertain *in each case* whether a *greater* or a *smaller* number is required, and must set down the greater or smaller number in the *middle* accordingly.

Then we must multiply together the first terms of the two statements for a common divisor; and the second terms of the two statements and the third term must be multiplied together for the dividend; after which we must proceed as *in the Simple* Rule of Three.

Thus having set down our third term 8, we must take the quantities of earth as our first and second terms; and as fewer men will be required to remove 200 yards than will be required to move 240, our middle term must be the *smaller* of the two. Then for the two first terms of the second statement we must take the periods of time; and as it will take more men to do a piece of work in 4 days than in 6 days, we must take the *greater* number for our middle term; and the statements will be made as follow.

```
As          240  :  200  ::  8
As            4  :    6  ::  —

        240             200
          4               6
        ———             ———
        960            1200
        ═══               8
                        ———
                 960)9600(10
                        960
                        ———
                       ...0
                        ═══
```

Here we multiply 240 and 4 together for a divisor, and 200, 8, and 6 together for a dividend; and we have a quotient 10, or find that 10 men will be required to do the work in the time required.

Again. If a person can travel 300 miles in 10 days, when the days are 12 hours long, how many days will he be travelling 800 miles, when the days are 16 hours long?

Here *days* are required for the answer, therefore 10 must be our third term. He will evidently require a *longer* time for a journey of 800 miles, than for a journey of 300 miles; therefore the *greater* number of the two must be in the middle, in our first line of the statement. But when the days are 16 hours long, he will require *fewer* days for his journey, than when they are only 12 hours long; therefore in the second line of the statement the smaller number of the two, or 12, must be the middle term, and the statement will be,

```
As     300  :  800  ::  10
As      16  :   12  ::  —

       300          800
        16           12
      ----         ----
      1800         9600
      300            10
      ----        -----
      4800   48|00)960|00(20
      ====         96 |
                  ----
                   ..0
                  ====
```

Multiplying together the two middle terms and the third term for a dividend and the two first terms for a divisor, and then dividing, we obtain a quotient 20, or find that 20 days will be required for the journey.

Exercise 95.

1. If a family of 18 persons spent 240*l.* in 8 months, how much will serve 40 persons for 18 months?

2. If 16 men can dig 48 yards of earth in 6 days, how many men must there be to dig 192 yards of earth in 4 days?

3. If the carriage of 5 cwt. 3 qrs. of goods for 300 miles cost 6*l.* 17*s.* 6*d.*, what will be the carriage of 7 cwt. 2 qrs. 18 lbs. for 250 miles?

4. If 960 slabs, each 12 inches square, will pave the floor of a building, how many slabs would be required for the same floor that were 10 inches long, and 7 inches broad?

5. If 327 persons can be maintained 7 days for 55*l.* 3*s.* 7½*d.*, what will be the cost of maintaining 186 persons for 40 days?

6. A corn-factor sent his correspondent 10,000 quarters of wheat at 8*l.* 7*s.* 6*d.* a load; he has received 10,000*l.* in return: how much is still due to him?

7. Having bought 4 bags of hops, weighing together 8 cwt. 3 qrs. 14 lb., at the rate of five guineas per cwt., what did they cost me?

8. Multiply 9 cwt. 3 qrs. 17 lb. by 27.

Sometimes *three statements* may be combined thus into *one;* but *this* will seldom be required, as the work may often be simplified by multiplying together, before the statement is made, such numbers as represent parts of the same thing.

If we have such a question as this: "If 300 men in 6 days can dig a trench 350 yards long, 4 yards wide, and 3 yards deep; in how many days can 50 men dig a trench 430 yards long, 5 yards wide, and 3 yards deep?" The length, breadth, and depth of the ditch may be multiplied together before the statement is made, and will give in a simple number the amount of work to be done.

Thus 350 × 4 × 3 give 4200 yards of earth removed; and 430 × 5 × 3 give 6450 yards, the quantity of earth to be removed, and the question is at once reduced to the following:

If 300 men in 6 days can remove 4200 yards of earth, in how many days can 50 men remove 6450 yards of earth? and the statement will be,

As 50 : 300 :: 6
As 4200 : 6450 :: —

4200
50
———
210000

6450
300
———
1935000
6
———
21|0000)1161|0000(55 days.
105
——
111
105
——
6
12
—
21)72(3 hrs.
63
—
9
60
——
21)540(25

21)540(25 min.
42
——
120
105
——
15

And we find that they can do it in 55 days, 3 hours, and nearly a half. In this sum the first remainder has been multiplied by 12, to reduce it to hours (there being 12 working hours only in a day); and the second remainder has been multiplied by 60, to reduce it to minutes.

Exercise 96.

1. If 9 horses consume 8 bushels of oats in 6 days, how much will 16 horses consume in a month?
2. If a barrel of beer will last a family of 12 persons 7 days, how many barrels will 10 persons drink in a year?
3. If the expense of 12 labourers for 18 weeks is 128*l.* 12*s.*, what will be the expense of 250 labourers for a month?
4. If 97 men can dig a trench 100 yards long, 5 yards wide, and 3 yards deep, in 4 days, in how many days will 24 men dig a trench 300 yards long, 6 yards wide, and 4 yards deep?
5. If an iron bar 6 feet long, 3 inches broad, and 2 inches deep, weigh 80 lbs., what will be the weight of a bar 20 feet long, 4 inches broad, and 3 inches deep?
6. If 5 men could dig a trench 430 yards long, 5 yards wide, and 3 yards deep, in 55 days 3½ hours; how long will it take 300 men to dig a trench 350 yards long, 4 yards wide, and 3 yards deep?
7. How many miles are there in 58,734,720 inches?
8. How many poles are there in 54 acres?

If the question were as follows:

If 300 men in 6 days, of 15 hours each, can dig a trench 350 yards long, 4 wide, and 3 deep; in how many days, of 8 hours each, can 50 men dig a trench 430 yards long, 5 wide, and 3 deep? then, even after the reduction which we made in the last example which was worked, we should be obliged to have three statements, for the question then becomes,—If 300 men in 6 days, of 15 hours each, can remove 4200 yards of earth; in how many days, of 8 hours each, can 50 men remove 6450 yards of earth? in which *there are still* seven numbers involved.

And our statement will be,

```
As     50 :  300 :: 6
As   4200 : 6450 :: —
As      8 :   15 :: —

     4200         6450
       50          300
   ------      -------
   210000      1935000
        8           15
  -------      -------
  1680000      9675000
  =======     1935000
              --------
              29025000
                     6
  168|0000)17415|0000(103 days.
           168
           ----
             615
             504
             ---
             111
               8
             ---
        168)888(5 hours.
            840
            ---
             48
             ===
```

And multiplying together all the second terms and the third term for a dividend, and all the first terms for a divisor, and dividing the former product by the latter, we obtain a quotient 103 days, 5¼ hours (nearly).

We multiplied the remainder in this case by 8, to reduce it to hours, because the days are to be only 8 hours long.

Exercise 97.

1. If 250 men in 5 days, of 9 hours each, can dig a trench 220 yards long, 3 wide, and 2 deep; in how many days, of 8 hours long, will 30 men dig a trench 450 yards long, 4 wide, and 3 deep?

2. If 50 men in 104 days, when the day is 8 hours long,

can dig a trench 430 yards long, 5 wide, and 3 deep, how many men can dig a trench 350 yards long, 4 wide, and 3 deep, in 6 days, when there are 15 hours to the day?

3. If a garrison of 800 soldiers have provisions for 28 days, at the rate of 17 ozs. per day, how long will their provisions last if their number be increased by a reinforcement of 250 men, and the daily allowance of each man is reduced to 15 ozs.?

4. If A can walk 300 miles in 8 days, when the day is 12 hours long, how long will he be walking 600 miles, when the day is 15 hours long?

5. If the cost of 14 horses for 24 days is 25*l.* 4*s.*, what will be the cost of a troop of 40 horses for 8 months?

6. If 96 English ells cost 16*l.* 10*s.*, what will 3 yards cost?

7. A draper bought 12 pieces of cloth, each containing 23 English ells, at the rate of 3*s.* 4*d.* for a Flemish ell; what was the cost?

8. Divide 33,521*l.* 16*s.* 3*d.* by 11.

PRACTICE

Is a convenient and easy method of working those sums in the Rule of Three which have a unit of any denomination for the first term.

When the first term in any Rule of Three sum is only 1, there is no necessity for dividing by that first term, and the operation is confined to multiplying together the second and third terms, so that in fact the sum is a sum in COMPOUND MULTIPLICATION.

And PRACTICE is the method of finding the answer by a shorter process than that of multiplication.

The name arises from the general use of the Rule: for it is *practised* by most men of business in finding quickly the value of any quantity of similar articles, from the known price of one of the articles.

Suppose we had to find the value of 2880 pounds of tea at 6*s.* 8*d.* per pound.

By the rule for Compound Multiplication we should have *to set down* 6*s.* 8*d.*, and multiply it by the several numbers, whose continued product is 2880, that is, by 8, 8, 5, and 9 —or 12, 12, 10 and 2.

Thus,

	s.	d.
	6	8
		8
2	13	4
		8
21	6	8
		5
106	13	4
		9
960	0	0

And we find that the cost of 2880 lbs. of any article at 6*s*. 8*d*. per lb. would be 960*l*.

Or the sum might be stated as a sum in the Rule of Three.

Thus,

	lb.		lb.		s.	d.
As,	1	:	2880	::	6	8
			80		12	
		12)	230400		80	
		2,0)	1920,0			
			960			

In which case we have to reduce 6*s*. 8*d*. to pence, then to multiply 2880 by it, and (as the answer will be in pence) to reduce the product to pounds. This method is perhaps shorter than the former.

But by Practice the work is very short indeed.

The sum 6*s*. 8*d*. is the third part ($\frac{1}{3}$) of a pound, so that by dividing 2880 by 3, we shall have the answer at once in pounds.

$$\text{Thus,}\quad \begin{array}{ll} \text{s. d.} & \text{£.} \\ 6\ \ 8 = \frac{1}{3} \text{ of } 1) & 2880 \\ \hline & 960 \end{array}$$

When therefore suitable divisors can be found, the work is speedily done.

In this method when the price is *shillings*, the answer is in *pounds*; when the price is *pence*, the answer is *shillings*, and must be reduced to pounds; when the price is *farthings*, the answer is *pence*, and must be reduced to pounds, and shillings, &c.

But the price of the article is sometimes such that it is not an exact divisor of the next higher denomination. In this case we shall have to use two divisors, or more.

Suppose the price had been 12 shillings instead of 6*s*. 8*d*., or we had to find the value of 2880 pounds of tea at 12*s*. each pound: we might find the answer easily in this case by multiplication, or by the Rule of Three; but it may easily be done by Practice.

12 shillings is not an exact divisor of 1*l*. or 20 shillings: but 12 = 10 + 2, and we may therefore find the cost at 10 shillings a pound, and the cost at 2 shillings a pound, and adding together the results, we shall have the cost at 12 shillings a pound.

$$\text{Thus,}\quad \begin{array}{r|r} 10s.\text{ is } \frac{1}{2} \text{ of } 1l. & 2880 \\ \hline 2s.\text{ is } \frac{1}{5} \text{ of } 10s. & 1440 \\ & 288 \\ \hline & 1728 \end{array}$$

10 shillings is half of 1*l*.; dividing therefore 2880 by 2, we obtain the value of 2880 pounds at 10*s*., in the denomination pounds, which equals 1440*l*.; and as 2*s*. is the ($\frac{1}{5}$) fifth part of 10 shillings, the cost of the same number of pounds at 2 shillings will be the fifth part of the cost at 10 shillings, or 1440*l*. divided by 5, which gives a quotient 288*l*.; and the two sums added together, or 1728*l*., will be the cost of 2880 lbs. at 10 + 2 shillings, or 12 shillings.

2*s*. is the *fifth* part ($\frac{1}{5}$) of 10 shillings; it is also the *tenth part* ($\frac{1}{10}$) *of* 20 shillings, or 1*l*.; so that it makes no difference whether we divide the cost at 10 shillings by 5, or the cost at 1*l.* by 10*s*; that is whether we divide the second line by 5, or the first line by 10; but it is most usual, when it can be done, to obtain each line from the line immediately above it by using the proper divisor for that line.

The numbers 10*s.*: 2*s.*: 6*s.* 8*d.*: which are exact divisors of 20 shillings, or 1*l.*, are called *aliquot parts* of a pound: and, in general, the aliquot parts of any number are such parts as, being taken a certain number of times, will exactly produce that number. Thus 3 is an aliquot part of 12, because 4 times 3 = 12: 5 is an aliquot part of 20, because 4 times 5 = 20: 3*s.* 4*d.* is an aliquot part of a pound, because 6 times 3*s.* 4*d.* = 20 shillings or 1*l.*; and 2*s.* 6*d.* is an aliquot part of a pound, because 8 times 2*s.* 6*d.* = 20 shillings, or 1*l.*

PRACTICE TABLES.

s.	d.		Of a Pound.
10	0	are	1 half.
6	8		1 third.
5	0		1 fourth.
4	0		1 fifth.
3	4		1 sixth.
2	6		1 eighth.
2	0		1 tenth.
1	8		1 twelfth.
1	0		1 twentieth.

d.		Of a Shilling.
6	are	1 half.
4		1 third.
3		1 fourth.
2		1 sixth.
1½		1 eighth.
1		1 twelfth.

cwt.		Of a Ton.
10	are	1 half.
5		1 fourth.
4		1 fifth.
2½		1 eighth.
2		1 tenth.

qrs.		lbs.		Of a Cwt.
2	or	56	are	1 half.
1	or	28		1 fourth.
		14		1 eighth.

lbs.		Of a Quarter.
14	are	1 half.
7		1 fourth.
4		1 seventh.
3½		1 eighth.

CASE 1.

When the price is less than a penny, take the aliquot parts of a penny, add together the quotients, and reduce the answer to pounds.

What is the cost of 2,141,567 at 3 farthings (¾d.)?

½d. is ½ of 1d.	2141567		
¼d. is ½ of ½d.	1070783½		
	535391¾		
12	1606175¼		
2,0	13384,7		11¼
	6692	7	11¼

Here we have to find the value at 3 farthings; now 3 farthings = 1 halfpenny and 1 farthing; we may therefore find the value at 1 halfpenny, and then the value at 1 farthing, and the two results added together will be the value at 3 farthings.

We say, therefore, 1 halfpenny (½d.) is the half (½) of 1 penny, and dividing 2,141,567 by 2, we have the cost of this number of articles at 1 halfpenny each; we then say one farthing (¼d.) is the half (½) of one halfpenny; and again dividing the last quotient by 2, we have the cost of the same number of articles at 1 farthing (¼d.) each; adding therefore together these two lines, we have the cost at 1 halfpenny, and the cost at 1 farthing, *i. e.* the cost at 3 farthings; and reducing the answer to pounds, we find the cost of 2,141,567 articles at 3 farthings each to be 6692*l.* 7*s.* 11¼*d.*

Again. What is the cost of 2147 articles at ¼d. each?

¼d. is ¼ of 1d.	2147		
12	536¾		
2,0	4,4		8¾
	2	4	8¾

Here we have only to take parts for 1 farthing, and we *say, 1 farthing* is the fourth part (¼) of 1 penny, and dividing 2147 by 4, we have an answer 536 pence and 3 farthings; and reducing this to pounds, we find the cost of 2147 articles, at ¼*d.* each, to be 2*l.* 4*s.* 8¾*d.*

Exercise 98.

1. What is the cost of 3082 buttons at ¼*d.* each?
2. What is the cost of 8847 articles at ½*d.* each?
3. What is the cost of 871,412 screws at ¾*d.* each?
4. What is the cost of 98,141 nails at ¼*d.* each?
5. What is the cost of 8,714,165 books at ¾*d.* each?
6. A draper bought 4 packs of cloth; each pack contained 3 parcels, and each parcel 6 pieces of cloth 40 yards in length. He gave 3*l.* 10*s.* for every 6 yards: what did they cost?
7. What will 19 shillings and 4 pence a day come to in 1 year?
8. If one ounce and a half of silver cost 10*s.* 10*d.*, what will be the cost of 29 lbs. 4 oz.?

CASE 2.

If the value consists of pence and farthings, we must begin with the pence as the aliquot parts of a shilling, and then proceed to the farthings, and our answer will be in *shillings;* which being divided by 20, will give the amount in pounds, shillings, &c.

What is the cost of 216 yards of calico at 4½*d.* a yard?

4*d.* is the third part (⅓) of a shilling, and a ½*d.* is the eighth part (⅛) of 4*d.*; so that in this case we shall have to divide 216 by 3, to find the value at 4*d.*, and the quotient so obtained by 8, to find the value at ¼*d.*

4*d.* = ⅓ of 1*s.*	216 at 4½*d.*
½*d.* = ⅛ of 4*d.*	72
	9
2,0)	8,1
	4 1

Here, dividing by 3, we find that the value of the calico at 4 pence is 72 shillings; then, as ½*d.* is the eighth part of 4 pence, we divide this quotient by 8, which gives us 9 shillings as the cost of the calico at 1 halfpenny, and adding the two results together, we find the whole cost at 4½*d.* to be 81 shillings, or 4*l.* 1*s.*

Observe: the first quotient (and therefore all the rest) is always one denomination higher than the highest denomination which is mentioned in the price given.

Again,

What is the cost of 6145 wine glasses at 5½*d.* each?

5 is not an aliquot part of 12; but 5 = 4 + 1, which are both aliquot parts of 12; we must therefore begin by finding the value at 4 pence, and then proceed to find the value at 1 penny, and the value at 1 halfpenny; and the sum of the three quotients will be the value at 4 pence + 1 penny + 1 halfpenny, or at 5½*d.*

Thus,

$$\begin{array}{r|rll}
4d. = \frac{1}{3} \text{ of } 1s. & 6145 & & \\
\cline{2-3}
1d. = \frac{1}{4} \text{ of } 4d. & 2048 & 4 & \\
\frac{1}{2}d. = \frac{1}{2} \text{ of } 1d. & 512 & 1 & \\
 & 256 & 0\frac{1}{2} & \\
\cline{2-3}
2{,}0 & 281{,}6 & 5\frac{1}{2} & \\
\cline{2-4}
\multicolumn{1}{r}{} & 140 & 16 & 5\frac{1}{2} \\
\cline{2-4}
\end{array}$$

Here we say 4 pence is one third part (⅓) of a shilling; then dividing by 3, we find the cost at 4*d.* to be 2048*s.* 4*d.*; and, as 1 penny is the fourth part (¼) of 4 pence, dividing this quotient by 4, we find the value at 1 penny to be 512*s.* 1*d.*: then, as ½*d.* is the half of 1 penny, dividing this second quotient by 2, we find the cost at ½*d.* to be 256*s.* 0½*d.*: we then add together the three quotients, and their sum, 2816*s.* 5½*d.*, is the cost at 4 pence, and 1 penny, and 1 halfpenny, that is, at 5 pence halfpenny: and reducing the shillings to pounds, we find the whole cost to be 140*l.* 16*s.* 5½*d.*

What is the cost of 2141 yards at 7¼*d.* per yard?

7 pence is not an aliquot part of 1 shilling, but 4 pence is so, and 6 pence is so, we may therefore consider 7 either as 6 and 1 or as 4 and 3.

	(1)	
6*d.* = ½ of 1*s.*	2141	
1*d.* = ⅙ of 6*d.*	1070	6
¼*d.* = ¼ of 1*d.*	178	5
	44	7¼
2,0	129,3	6¼
	64 13	6¼

	(2)	
4 = ⅓ of 1*s.*	2141	
2 = ½ of 4*d.*	713	8
1 = ½ of 2*d.*	356	10
¼ = ¼ of 1*d.*	178	5
	44	7¼
2,0	129,3	6¼
	64 13	6¼

Here we have in the first case considered 7 pence as 6 + 1 pence, and in the second case as 4 + 2 + 1 pence, and we find the results to be the same. In the first case our divisors are 2, 6 and 4, because 6 pence is the half (½) of a shilling, 1 penny is the sixth (⅙) of 6 pence, and 1 farthing is the fourth (¼) of 1 penny. In the second case we have for divisors 3, 2, 2 and 4, because 4 pence is the third part (⅓) of 1 shilling, 2 pence is the half (½) of 4 pence, 1 penny is the half (½) of 2 pence, and 1 farthing is the fourth part (¼) of 1 penny.

Exercise 99.

1. What is the cost of 381,412 loaves of bread at 3½*d.* each?
2. What is the cost of 91,416 yards of print at 5¼*d.* per yard?
3. What is the cost of 214,714 oranges at 1½*d.* each?
4. What is the cost of 81,714,125 yards of print at 9¼*d.* per yard?
5. What is the cost of 214,168,754 yards of calico at 7¾*d.* per yard?
6. What is the cost of 916,141,516 yards of cambric at 10¾*d.* per yard?
7. What is the value of 18 packages of figs, each weighing 3 qrs. 18 lb., at the rate of 6*s.* 6*d.* for every 12 lbs.?

8. The rateable value of a parish is 1750*l.*, and a rate is granted of 32*l.* 16*s.* 3*d.*: what is it in the pound?

9. Divide 614,159*l.* 10*s.* 6*d.* by 84.

Exercise 100.

1. What is the cost of 81,415 loaves of bread at 8¾*d.*?

2. What is the cost of 98,276 lbs. of mutton at 4½*d.* per lb.?

3. What is the cost of 71,471,462 yards of muslin at 9¾*d.* per yard?

4. What is the cost of 614,149 yards at 3¾*d.* per yard?

5. What is the cost of 123,456,789 at 6½*d.*?

6. What is the cost of 8,147,654,321 gallons of ale at 11¾*d.* per gallon?

7. A bankrupt is indebted 2980*l.* 10*s.*; his effects amount to 931*l.* 8*s.* 1½*d.*; what will his creditors receive in the pound?

8. Having bought a cargo of coals consisting of 33 chaldron 12 bushels for 50*l.*, what did they cost me per bushel?

9. From 340 lb. 6 oz., take 265 lb. 8 oz. 9 dwt. 15 grs.

CASE 3.

When the price, or value, of each article consists of shillings, or of shillings and pence, or of shillings, pence and farthings, we must begin with the shillings as aliquot parts of a pound, and then proceed to the pence and farthings; and, adding together all the quotients, we shall have the whole value in pounds, shillings, &c. &c.

What is the value of 2146 articles at 3*s.* 6½*d.* each?

3*s.* 6½*d.* is not itself an aliquot part of 1*l.*; we must therefore take the aliquot part of 1*l.* nearest to this sum, and this will be 3*s.* 4*d.*, which is the sixth part (⅙) of 1*l.*; and we shall have 2½*d.* remaining, for which we must take parts afterwards.

3*s.* 4*d.* = ⅙ of 1*l.*	2146		
2*d.* = $\frac{1}{20}$ of 3*s.* 4*d.*	357	13	4
½*d.* = ¼ of 2*d.*	17	17	8
	4	9	5
	380	0	5

Here we begin with 3*s*. 4*d*., which is the sixth part ($\frac{1}{6}$) of 1*l*., and dividing by 6, we have a quotient 357 pounds, and a remainder 4*l*.; reducing this remainder to shillings, and dividing by 6, we have a quotient 13 shillings, and a remainder 2: this we reduce to pence; and again dividing by 6, we have a quotient 4 for the pence.

Then taking parts for the 2 pence, it is the twentieth part ($\frac{1}{20}$) of 3*s*. 4*d*. (which = 40 pence), and dividing the last quotient by 20, we obtain a quotient 17*l*. 17*s*. 8*d*. as the value at 2 pence. We then have to take parts for 1 halfpenny. This is the fourth part ($\frac{1}{4}$) of 2 pence: and dividing by 4, we have a quotient 4*l*. 9*s*. 5*d*., as the value at $\frac{1}{2}$*d*.; and adding the three quotients together, we find the value of 2146 articles, at 3*s*. 6$\frac{1}{2}$*d*. each, to be 380*l*. 0*s*. 5*d*.

We might have begun by taking 2*s*. 6*d*. as the eighth part ($\frac{1}{8}$) of a pound, and then have taken parts for 1*s*. 0$\frac{1}{2}$*d*., which remain from 3*s*. 6$\frac{1}{2}$*d*.

Thus, 2*s*. 6*d*. = $\frac{1}{8}$ of 1*l*.	2146		
10*d*. = $\frac{1}{3}$ of 2*s*. 6*d*.	268	5	
2*d*. = $\frac{1}{5}$ of 10*d*.	89	8	4
$\frac{1}{2}$*d*. = $\frac{1}{4}$ of 2*d*.	17	17	8
	4	9	5
	380	0	5

In this case we begin with 2*s*. 6*d*., which is the eighth part ($\frac{1}{8}$) of 1*l*., and, dividing 2146 by 8, we have a quotient 268*l*. 5*s*.; we then take 10 pence, which is the third part ($\frac{1}{3}$) of 2*s*. 6*d*. (or 30 pence), and dividing the last quotient by 3, we obtain a quotient 89*l*. 8*s*. 4*d*. We have now to take parts for 2 pence, which is the fifth part ($\frac{1}{5}$) of 10 pence; and dividing the second quotient by 5, we have a quotient 17*l*. 17*s*. 8*d*. There still remains 1 halfpenny; this is the fourth part ($\frac{1}{4}$) of 2 pence, and dividing the last quotient by 4, we have a quotient 4*l*. 9*s*. 5*d*.: then adding together these four quotients, we have the value at 2*s*. 6*d*. 10*d*., 2*d*., and $\frac{1}{2}$*d*., or at 3*s*. 6$\frac{1}{2}$*d*. And this is, as before, 380*l*. 0*s*. 5*d*.

What is the value of 281,413 writing desks at 15*s*. 8$\frac{1}{2}$*d*. each?

In this case it will be most convenient to divide the 15 into 10 and 5.

10*s*. = ½ of 1*l*.	281413		
5*s*. = ½ of 10*s*.	140706	10	
6*d*. = $\frac{1}{10}$ of 5*s*.	70353	5	
2*d*. = ⅓ of 6*d*.	7035	6	6
½*d*. = ¼ of 2*d*.	2345	2	2
	586	5	6½
	221026	9	2½

Here we have 10 shillings, equals the half of 1 pound, and dividing by 2, we have a quotient 140,706*l*. 10*s*. Then we have 5 shillings, equals the half of 10 shillings, and dividing 140,706*l*. 10*s*. by 2, we have a quotient 70,353*l*. 5*s*. We then have 6 pence, the tenth part of 5 shillings, and dividing 70,353*l*. 5*s*. by 10, we have a quotient 7035*l*. 6*s*. 6*d*. We then have 2 pence, the third part of 6 pence, and dividing 7035*l*. 6*s*. 6*d*. by 3, we have a quotient 2345*l*. 2*s*. 2*d*. We have, lastly, 1 halfpenny, the fourth part of 2 pence, and dividing 2345*l*. 2*s*. 2*d*. by 4, we have a quotient 586*l*. 5*s*. 6½*d*. And adding together the several quotients, we have the whole value at 15*s*. 8½*d*. which is 221,026*l*. 9*s*. 2½*d*.

Exercise 101.

1. What is the value of 2146 at 6*s*. 10½*d*.?
2. What is the value of 31,415 planes at 7*s*. 6*d*. each?
3. What is the value of 812,134 dresses at 8*s*. 4*d*. each?
4. What is the value of 6,141,476 dresses at 9*s*. 10½*d*. each?
5. What is the value of 2,714,176 hats at 14*s*. 6*d*. each?
6. What is the value of 9,141,476 pairs of shoes at 16*s*. 5¾*d*. a pair?
7. What will be the cost of a silver waiter, weighing 4 lb. 11 oz. 10 dwts., when silver is worth 5*s*. 4*d*. an ounce?
8. What interest would 631*l*. 5*s*. yield in a year, if 100*l*. yielded in the same time 3*l*.?
9. If I lend 100*l*. for a year, and receive for the use of

it 3*l.* 10*s.*, how much ought I to receive for the loan of 360*l.* for 3 years?

Exercise 102.

1. What is the cost of 8141 hats, at 16*s.* 4*d.* each?
2. What is the cost of 81,713 hats, at 18*s.* 6½*d.* each?
3. What is the cost of 271,415 cloaks, at 15*s.* 10*d.* each?
4. What is the cost of 2,816,144 pairs of shoes, at 12*s.* 11¼*d.* per pair?
5. What is the cost of 314,178 loaves of bread, at 6¾*d.*?
6. What is the cost of 91,416 pounds of sago, at 7½*d.* a pound?
7. If 10 men consume 450 lbs. of beef in 3 weeks, how much will 436 men consume in 10 weeks?
8. If I can walk 33 miles in a day, when the day is 14 hours long, how many miles can I walk in a week, when the day is 16 hours long?
9. What is the cost of 1753 oranges, at ¾*d.* each?

Case 4.

When the value of each article is greater than one pound, we must first multiply the number by the number of pounds, and then take parts for the shillings and pence by the last rule.

In this case, however, we must remember that the division by the aliquot parts must begin with the *first line*, and *not* with the product of that line multiplied by the pounds.

What is the value of 2146, at 2*l.* 6*s.* 8*d.*?

6*s.* 8*d.* = ⅓ of 1*l.*	2146		
	2		
	4292		
	715	6	8
	5007	6	8

Here we have 2146 articles, at 2*l.* each; and we find their value in pounds by multiplying by 2. We have then to find the value of the same number of articles, at 6*s.* 8*d.*

and as this is the third part of 1*l.* we shall obtain the value by dividing 2146 by 3, and setting this quotient under the product just found, and adding the two lines together, we find the value of 2146 articles, at 2*l.* 6*s.* 8*d.*, to be 5007*l.* 6*s.* 8*d.*

What is the value of 314,159 articles, at 3*l.* 11*s.* 11*d.* each?

10*s.* = $\frac{1}{2}$ of 1*l.*	314159		
	3		
	942477		
1*s.* = $\frac{1}{10}$ of 10*s.*	157079	10	
6*d.* = $\frac{1}{2}$ of 1*s.*	15707	19	
3*d.* = $\frac{1}{2}$ of 6*d.*	7853	19	6
1*d.* = $\frac{1}{3}$ of 3*d.*	3926	19	9
1*d.* = 1*d.*	1308	19	11
	1308	19	11
	1129663	8	1

We first multiply by 3, for the cost at 3*l.*; then we divide the 11 shillings into 10 and 1; and the 11 pence into 6, 3, 1, and 1, and our divisors are 2, 10, 2, 2, and 3; for the last penny we have no division, as it is *no part* of the preceding aliquot part, but is equal to it; we therefore set down the same sum a second time.

We might have taken parts for 12 shillings, and then have found the value at 1 penny, and subtracted it from the whole sum found for 3*l.* 12*s.*: as 11*s.* 11*d.* is equal to 12 shillings, all but (or —) one penny.

Thus,

10*s.* = $\frac{1}{2}$ of 1*l.*	314159			
	3			
	942477			
2*s.* = $\frac{1}{5}$ of 10*s.*	157079	10		
1*d.* = $\frac{1}{24}$ of 2*s.*	31415	18		
	1130972	8		value at 3*l.* 12*s.*
	1308	19	11	value at 1 penny.
	1129663	8	1	——— 3*l.* 11*s.* 11*d.*

Exercise 103.

1. What is the cost of 8,147,146 articles, at 2*l.* 10*s.* 10*d.* each?

2. What is the cost of 617,141 head of cattle, at 8*l.* 14*s.* 6*d.* each?

3. What is the cost of 91,414 cart horses, at 16*l.* 8*s.* 11½*d.* each?

4. What is the cost of 614,147 paintings, at 7*l.* 4*s.* 6½*d.* each?

5. What is the value of 98,276 dresses, at 16*s.* 7¾*d.* each?

6. What is the value of 61,414,729 articles, at 11½*d.* each?

7. A merchant shipped for Jamaica 500 pair of stockings, at 3*s.* 6*d.* a pair, and 1650 yards of baize, at 1*s.* 3*d.* a yard; and he received in return 348 gallons of rum, at 6*s.* 8*d.* a gallon, and 750 lbs. of indigo, at 1*s.* 4*d.* a pound: what remains due to him?

8. How far can a pedestrian walk in 7 days 9 hours 30 minutes, whose pace is 4 miles 3 furlongs in one hour?

Exercise 104.

1. What is the value of 814,176 loads of hay, at 12*l.* 14*s.* 10*d.* each?

2. What is the value of 914,762 articles, at 18*s.* 6½*d.* each?

3. What is the value of 37,141 horses, at 14*l.* 7*s.* 10¾*d.* each?

4. What is the cost of 91,814,729 yards of linen, at 9¾*d.* a yard?

5. What is the cost of 217,695 dresses, at 19*s.* 11¾*d.* each?

6. Wishing to ascertain the height of a church, I measured its shadow, and found that to be 75 yards. I then set up a stick which was 3 feet long, and found its shadow to be 5 feet; what was the height of the church?

7. How long would Balbus be building a wall 100 yards long, 4 feet thick, and 20 feet high; if he could complete a piece 3 yards long, 1 foot thick, and 5 feet high, in a day?

8. What is the amount of commission on 245*l.* 6*s.* 10*d.*, at 2½ per cent.?

CASE 5.

When the number consists of several denominations, as hundredweights, quarters, &c.; and the price given is the *price of one unit of the highest denomination*, we must set down, not the number, but the *price*; and multiply it by the number of units in the highest denomination, and take parts for the lower denominations.

What is the cost of 3 cwt. 2 qrs. 7 lbs., at 34*l.* 4*s.* 6*d.* per cwt.?

	£.	*s.*	*d.*
2 qrs. = $\frac{1}{2}$ of 1 cwt.	34	4	6
			3
	102	13	6
7 lb. = $\frac{1}{8}$ of 2 qrs.	17	2	3
	2	2	$9\frac{1}{4}$
	£121	18	$6\frac{1}{4}$

Here 34*l.* 4*s.* 6*d.* is the price of 1 hundredweight, therefore multiplying this sum by 3, we have the cost of 3 hundredweight: 2 quarters = $\frac{1}{2}$ an hundredweight; therefore the price of 2 quarters is half the price of one hundredweight, or the half of 34*l.* 4*s.* 6*d.*: we therefore divide this sum by 2, and find the value of 2 quarters to be 17*l.* 2*s.* 3*d.* Now 7 lb. is the eighth part ($\frac{1}{8}$) of 2 quarters (or 56 lb.); dividing therefore the value of 2 quarters by 8, we obtain 2*l.* 2*s.* $9\frac{1}{4}$*d.* (with a small remainder) as the value of 7 lb.; and the sum of these values, or 121*l.* 18*s.* $6\frac{1}{4}$*d.*, is the answer required.

What is the value of 8 ounces 16 pennyweights of silver, at 5*s.* 6*d.* per ounce?

		s.	*d.*
10 dwt. = $\frac{1}{2}$ an oz.		5	6
			8
	2	4	0
5 dwt. = $\frac{1}{2}$ of 10		2	9
1 dwt. = $\frac{1}{5}$ of 5		1	$4\frac{1}{2}$
			$3\frac{1}{4}$ nearly.
	2	8	$4\frac{3}{4}$

Here we multiply the price of one ounce by 8, and thus find the value of 8 ounces to be 2*l.* 4*s.* We then take 10 dwt., which = half an ounce, and dividing 5*s.* 6*d.* by 2, find the value of 10 dwt. to be 2*s.* 9*d.*: now 5 dwt. is the half of 10 dwt.; therefore its value will be the half of 2*s.* 9*d.*, or 1*s.* $4\frac{1}{2}$*d.*; and 1 dwt. is the fifth part of 5 dwt.; therefore the value of 1 dwt. is 1*s.* $4\frac{1}{2}$*d.* divided by 5, or $3\frac{1}{4}$*d.* (nearly.) And the whole value is the sum of these values of the different parts, and equals 2*l.* 8*s.* $4\frac{3}{4}$*d.*

Exercise 105.

1. What is the carriage of 8 cwt. 3 qrs. of goods, at 2*l.* 10*s.* 6*d.* per cwt.?
2. What is the value of 11 oz. 17 dwt. 10 grs. of silver, at 5*s.* 9*d.* an ounce?
3. What will be the cost of 12 cwt. 2 qrs. 17 lb. of tea, at 29*l.* 10*s.* 6*d.* per cwt.?
4. What will be the cost of 76 cwt. 2 qrs. of sugar, at 5*l.* 12*s.* $6\frac{1}{2}$*d.* per cwt.?
5. What is the value of 19 tod, 1 stone of wool, at 3*l.* 12*s* 4*d.* per tod?
6. What is the cost of seven thousand, four hundred, and thirty-six oranges, at the rate of 5 for $4\frac{1}{2}$*d.*?
7. From 196 cwt. 3 qrs. 8 lb., take 194 cwt. 1 qr. 18 lb. 9 oz. 3 drs.
8. If 500*l.* were given to A, B, and C, so that A was to have 3 times, and B twice as much as C, what would each have?

Exercise 106.

1. What is the value of 130 yards 3 qrs. of cloth, at 18*s.* $6\frac{1}{2}$*d.* a yard?
2. What is the value of 360 acres, 3 roods, 4 poles, at 2*l.* 10*s.* 6*d.* an acre?
3. What is the value of 16 gals. 3 qts. of brandy, at 1*l.* 1*s.* $6\frac{1}{2}$*d.* a gallon?
4. What is the value of 8 cwt. 2 qrs. 11 lb. of cheese, at 4*l.* 8*s.* $4\frac{1}{2}$*d.* per cwt.?
5. What is the hire of a carriage for 9 months 22 days, at 12*l.* 12*s.* a month?

6. How many yards of shalloon, 3 qrs. wide, will line 4 yards of cloth which is 5 quarters wide?

7. If the 6*d.* loaf weighs 4 lb. 2 oz. when wheat is 3*s.* 6*d.* a bushel, what ought it to weigh when wheat is 5*s.* a bushel?

8. If 800 men in 3 months consume 10,000 loaves of bread, how many men will consume 50,000 loaves in 5 months?

TARE AND TRET.

In transacting business certain allowances are made by tradesmen and merchants.

Tare is an allowance made to the buyer for the weight of the package, box, barrel, bag, &c., which contains the goods purchased; and is generally an allowance of a certain weight upon each box, &c., or a certain rate upon each hundredweight, or a certain rate upon the whole weight.

Tret is an allowance made for waste, dust, &c. This allowance is generally 4 lb. in every 104 lb. or the twenty-sixth part ($\frac{1}{26}$) of the whole.

Cloff is a further allowance, after Tare and Tret have been deducted, in order that the weight may hold good when the goods are sold by retail. This allowance is generally 2 lb. in every 3 cwt. or the one hundred and sixty-eighth part ($\frac{1}{168}$) of the whole weight.

Gross Weight is the whole weight of the goods, together with the box, barrel, or bag, &c. which contains them.

Neat Weight is the weight remaining after all the allowances have been made.

Suttle is a term applied to each remainder. Thus after Tare has been deducted, the remainder is called **Tare Suttle**: after Tret has been deducted, the remainder is called **Tret Suttle.**

Allowance for Tret and Cloff is now but seldom made.

It is evident that these allowances or deductions, after their exact value has been ascertained, must be subtracted *from the gross weight.*

The allowances or deductions themselves will have to be ascertained in various ways, according to their nature. They can, however, all be easily ascertained by the rules through which we have already passed. And the various examples under this rule will be varied combinations of Compound Multiplication, Division, Addition, Subtraction, &c.

CASE 1.

When the tare is an allowance upon the whole weight, it must be subtracted from the gross weight, and the remainder will be the neat weight.

Example :

What is the neat weight of 38 boxes of raisins, each weighing 3 qrs. 15 lb., allowing 3 qrs. 17 lb. for tare ?

	qrs.	lbs.	
	3	15	
		6	× 6 + 2 = 38
5	1	6	
		6	
31	3	8	
1	3	2	
33	2	10	gross weight.
	3	17	tare.
32	2	21	neat weight.

Here we first find the gross weight by multiplying 3 qrs. 15 lb. by 38, or 6 times 6 plus 2 (see page 69), which gives a result 33 cwt. 2 qrs. 10 lb. And, subtracting from this the tare, 3 qrs. 17 lb., we find the neat weight to be 32 cwt. 2 qrs. 21 lb.

Exercise 107.

1. What is the neat weight of 19 cwt. 3 qrs. 14 lb., tare being 2 cwt. 1 qr. 20 lb. ?

2. What is the neat weight of 28 barrels of indigo, weighing 5 cwt. 1 qr. 2 lb. each, tare being 4 cwt. 3 qrs. 9 lb.?

3. What is the neat weight of 27 chests of tea, each weighing 5 cwt. 2 qrs. 7 lb., tare being 5 cwt. 0 qrs. 11 lb.?

4. The tare allowed on 3 packages, weighing respectively 4 cwt. 2 qrs. 9 lb., 7 cwt. 1 qr. 11 lb., and 5 cwt. 0 qrs. 18 lb., was 1 cwt. 2 qrs. 8 lb., what was the neat weight?

5. If 100*l.* gain 6*l.* interest in 12 months, in what time would 75*l.* gain 3*l.* interest?

6. What is the cost of 5000 yards of linen, at 1*s.* 11½*d.* per yard?

CASE 2.

When the tare is an *uniform allowance* upon each package, it must be multiplied by the number of packages, and the *product* must be subtracted from the gross weight.

What is the neat weight of 24 barrels of figs each weighing 3 qrs. 19 lb., the tare being 8 lb. per barrel?

```
      qrs.  lb.
       3    19                            24
             4 × 6 = 24                    8
  -------------                          -----
   3   2    20                     ⎧4 |  192 lb.
             6               28 =  ⎨  |  -----
  -------------                    ⎩7 |   48
  22   0     8 gross weight.        4 | ---------
   1   2    24 tare.                  |   6   24
  -------------                       | -------------
  20   1    12 neat weight.           |   1   2   24
  =============                       ==============
```

In this case we multiply 3 qrs. 19 lb., the weight of each barrel, by 24, (or 4 times 6,) the number of barrels, which gives a product 22 cwt. 0 qrs. 8 lb. for gross weight. We then multiply 8 lb., the tare on each barrel, by 24; which give a product 192 lb. or 1 cwt. 2 qrs. 24 lb., for the gross tare; and, subtracting this product from the gross weight, we find that the remainder, or neat weight, is *20 cwt. 1 qr. 12* lb.

In this case we might have deducted the 8 lb., tare on each barrel, from the gross weight of the barrel, and then have multiplied the result by 24, and the product would have given us the same final result.

Thus,

	qrs.	lb.	
	3	19	
		8	
	3	11	
		4	× 6 = 24
3	1	16	
		6	
20	1	12	

Here we subtract 8 lb. the tare on each barrel, from 3 qrs. 19 lb. the gross weight of each barrel; which leaves a remainder 3 qrs. 11 lb. for the neat weight of each barrel, and multiplying this quantity by 24, (or 4 times 6,) we obtain a product, 20 cwt. 1 qr. 12 lb. for the neat weight of the whole.

Exercise 108.

1. What is the neat weight of 4 packages, weighing 4 cwt. 1 qr. 7 lb. each, tare being 6 lb. per package?
2. A grocer received 8 packages of tea, each weighing 5 cwt. 2 qrs. 9 lb., what was the neat weight, allowing 11 lb. per package for tare?
3. What is the neat weight of 240 barrels of figs, each weighing 3 qrs. 19 lb., tare being 10 lb. per barrel?
4. What is the neat weight of 7 packages of goods, weighing in all 27 cwt. 1 qr. 6 lb., tare being 3 qrs. 6 lb. per package?
5. What is the cost of 280 sheep, at 1*l.* 12*s.* 6*d.* each?
6. Multiply 18*s.* 6*d.* by 64.

CASE 3.

When the allowance for tare is *different* on the different packages, the gross amount of tare must be subtracted from the gross weight of the packages.

What is the neat weight of 4 cwt. 3 qrs. 11 lb., tare being 17 lb.; 5 cwt. 1 qr. 8 lb., tare being 18 lb.; and 7 cwt. 2 qr. 9 lb., tare being 27 lb.?

cwt.	qr.	lb.	
4	3	11	gross.
5	1	8	
7	2	9	
17	3	0	gross.
	2	6	tare.
17	0	22	neat weight.

lb.	
17	tare.
18	
27	
62	tare.

28) 62 (2 qrs. 6 lb.
56
—
6

Here we find by addition, the gross weight of all the packages to be 17 cwt. 3 qrs. 0 lb.; and the gross weight of the tare to be 62 lb., or 2 qrs. 6 lb.; and, subtracting the latter quantity from the former, we find the neat weight to be 17 cwt. 0 qrs. 22 lb.

The result would have been the same, if we had found the neat weight of each package separately, and then added together the several results.

Thus,

cwt.	qr.	lb.	
4	3	11	gross.
		17	tare.
4	2	22	neat.

cwt.	qr.	lb.	
5	1	8	gross.
		18	tare.
5	0	18	neat.

cwt.	qr.	lb.	
7	2	9	gross.
		27	tare.
7	1	10	neat.

cwt.	qr.	lb.
4	2	22
5	0	18
7	1	10
17	0	22

Exercise 109.

1. What is the neat weight of 4 casks of sugar, weighing respectively 2 cwt. 1 qr. 7 lb.; 4 cwt. 2 qrs. 9 lb.; 5 cwt. 1 qr. 14 lb.; and 4 cwt. 2 qrs. 11 lb.; the tare being 8 lb.

on the first, 16½ lb. on the second, 20 lb. on the third, and 16 lb. on the fourth?

2. Find the neat weight of 4 casks of raisins, weighing respectively 3 cwt. 1 qr. 7 lb.; 3 cwt. 1 qr. 16 lb.; 2 cwt. 3 qrs. 27 lb.; and 3 cwt, 0 qrs. 5 lb.; the tare being 7 lb. on each cask.

3. What is the neat weight of 5 casks of sugar, weighing in all 24 cwt. 1 qr. 6 lb.; tare being 2 cwt. 2 qrs. 17 lb.?

4. Find the neat weight of the following packages: (1.) 4 cwt. 1 qr. 6 lb., tare 6 lb.; (2.) 5 cwt. 3 qrs. 11 lb., tare 8 lb.; (3.) 7 cwt. 1 qr. 15 lb., tare 11 lb.; (4.) 5 cwt. 1 qr. 21 lb., tare 9 lb.

5. What is the cost of 37 cwt. 2 qrs. 14 lb., at 7*l.* 10*s.* 9*d.* per cwt.?

6. What is the value of 14 ingots of silver, weighing 7 lb. 5 oz. 10 dwts. each, silver being worth 5*s.* an ounce?

Case 4.

When the allowance for tare is *a certain rate on each hundredweight*, we must find the value by the Rules for Practice, Case 5, (p. 124,) and subtract the value so found from the gross weight; the result will be the neat weight.

What is the neat weight of 215 cwt. 3 qrs. 21 lb., tare being allowed at the rate of 16 lb. per cwt.?

	cwt.	qr.	lb.	
16 lb. = $\frac{1}{7}$ of 1 cwt.	215	3	21	gross weight.
	30	3	11	tare.
	185	0	10	neat weight.

Here we take parts for the 16 lb., which is the *one-seventh part* ($\frac{1}{7}$) of an hundredweight: and dividing by 7 we find that the gross tare is 30 cwt. 3 qrs. 11 lb; and subtracting this from 215 cwt. 3 qrs. 21 lb., the gross weight, we find the remainder, or neat weight, to be 185 cwt. 0 qrs. 10 lb.

Exercise 110.

1. What is the neat weight of 8 barrels of potash, each weighing 1 cwt. 3 qrs. 5 lb., tare being 10½ lb. per cwt.?

2. A grocer received goods weighing 180 cwt. 1 qr. 11 lb., tare being allowed at 16 lb. per cwt. What was the neat weight of the whole?

3. What is the value of 12 hogsheads of sugar, at 4*l.* 10*s.* 0*d.* per cwt., each weighing 4 cwt. 3 qrs., tare being allowed at the rate of 14 lb. per cwt.?

4. What is the cost of 9 boxes of fruit, weighing 46 cwt. 2 qrs. 11 lb., at 5*l.* 7*s.* 6*d.* per cwt., tare being allowed at 21 lb. per box?

5. If 27 yards of cloth cost 4*l.* 14*s.* 6*d.*, what will be the cost of 17 yards?

6. What is the cost of 240 lb. of sugar, at 7½*d.* per lb., tare being 17 lb.?

When an allowance has to be made for tret as well as for tare, the tare must first be found by the preceding rules, and subtracted from the gross weight. The remainder is called tare suttle. This remainder divided by 26 will give the amount of tret: which, being subtracted from the tare suttle, will give the neat weight.

1. What is the neat weight of 98 cwt. 1 qr. 18 lb., tare being 2 cwt. 3 qr. 2 lb., and tret being 4 lb. in every 104 lb.?

	cwt.	qrs.	lb.	
	98	1	18	gross weight.
	2	3	2	tare.
4 lb. = $\frac{1}{26}$ of 104	95	2	16	suttle.
	3	2	20	tret.
	91	3	24	neat weight.

Here we subtract the tare, 2 cwt. 3 qrs. 2 lb., from the gross weight, and find the remainder, or suttle, to be 95 cwt. 2 qrs. 16 lb.; and dividing this by 26 (according to the rule, p. 76), we find the tret to be 3 cwt. 2 qrs. 20 lb.; and, subtracting this from the tare suttle, we have a re*mainder 91 cwt.* 3 qrs. 24 lb. for the neat weight.

2. What is the neat weight of 8 boxes of goods, each weighing 3 cwt. 1 qr. 5 lb., tare being 17½ lb. per cwt., and tret as usual?

		cwt.	qrs.	lb.		
		3	1	5		
				8		
14 lb. = $\frac{1}{8}$ cwt.		26	1	12		gross weight.
3½ = $\frac{1}{4}$ of 14.		3	1	5		
		0	3	8	4	
		4	0	13	4	tare.
	26)	22	0	26	12	suttle.
			3	11	12	tret.
		21	1	15	0	neat weight.

Here we multiply 3 cwt. 1 qr. 5 lb., the weight of each box, by 8, to find the gross weight, which is 26 cwt. 1 qr. 12 lb. The tare on this at 17½ lb. per cwt., we find to be 4 cwt. 0 qrs. 13 lb. 4 oz.; subtracting this from the gross weight, we have a remainder, or SUTTLE, 22 cwt. 0 qrs. 26 lb. 12 oz.; dividing this by 26, we find the tret to be 3 qrs. 11 lb. 12 oz., and subtracting this latter weight from the suttle 22 cwt. 0 qrs. 26 lb. 12 oz., we have a remainder 21 cwt. 1 qr. 15 lb. for the neat weight.

Exercise 111.

1. What is the neat weight of 8 cwt. 1 qr. 11 lb., tare being 3 qrs. 8 lb., and tret as usual?
2. What is the amount of tret on 127 cwt. 1 qr. 11 lb., tare being 5 cwt. 2 qrs. 17 lb.?
3. What will be the neat weight of 11 barrels of sugar, each weighing 5 cwt. 2 qrs. 7 lb., tare being 1 qr. 5 lb. per barrel, and tret as usual?
4. Find the difference between the tare and tret on 11 cwt. 2 qrs. 17 lb., tare being 14 lb. per cwt., and tret as usual.

5. What is the value of the allowance for tare and tret, at 4*l.* 10*s.* 0*d.* per cwt., on 5 packages of 2 cwt. 1 qr. 6 lb. each, tare being 14 lb. per package?

6. What is the tare on 136 cwt. 2 qrs. 19 lb., at 16 lb. per cwt.?

When cloff is allowed, we must deduct the tare and tret, as in the preceding rule, and then divide the remainder, or tret suttle, by 168, or its component numbers 4 × 6 × 7. This will give the amount of cloff; and, subtracting this from the tret suttle, we obtain the neat weight.

Let it be required to find the neat weight of 6 boxes of raisins, each weighing 3 cwt. 1 qr. 5 lb., tare being 17½ lb. per cwt., and tret and cloff as usual.

The weight of each box, and the rate per cwt. for the tare are the same in this example as in the former, but the number of boxes is 6 instead of 8.

	cwt.	qrs.	lb.		
	3	1	5		
			6		
14 lb. = $\frac{1}{8}$ cwt.	19	3	2		gross.
3½ = ¼ of 14.	2	1	24	12	
	0	2	13	3	
	3	0	9	15	tare.
26	16	2	20	1	tare suttle.
		2	15	13	tret.
168	16	0	4	4	tret suttle.
			10	11	cloff.
	15	3	22	9	neat weight.

		cwt.	qr.	lb.	oz.
168	4	16	0	4	4
	6	4	0	1	1
	7	0	2	18	13
		0	0	10	11

Here we multiply the weight of each box by 6, the number of boxes; and find the gross weight to be 19 cwt. 3 qrs. 2 lb. Then proceeding as in the last example, we find the tare and tret; and subtracting these in their respective order, we have a remainder 16 cwt. 0 qr. 4 lb. 4 oz., which we call tret suttle; dividing this by 168, or by 4 × 6 × 7, we obtain a quotient 10 lb. 11 oz. for cloff; and subtracting this from the tret suttle, we find the remainder, or neat weight to be, 15 cwt. 3 qrs. 22 lb. 9 oz.

A simple way of finding the cloff, and sufficiently accurate for all practical purposes, is to multiply the hundredweights in the tret suttle by 2, and divide the product by 3. The quotient will be the value of the cloff in pounds.

Thus, in the last Example in which the tret suttle was 16 cwt. 0 qrs. 4 lb. 4 oz.

```
    16
     2
    --
 3) 32
    ------
    10  10
    ======
```

We multiply the 16 by 2, and divide the product 32 by 3, which gives a quotient, 10 lb. 10 oz.

The reason of this is plain. The allowance for cloff is 2 lb. for every 3 cwt.; or two-thirds ($\frac{2}{3}$) of a pound for every hundredweight.

Exercise 112.

1. What is the neat weight of 27 parcels, each weighing 3 cwt. 0 qr. 14 lb., tare being 8 lb. per cwt., and tret and cloff as usual?
2. What is the neat weight of 256 cwt. 1 qr. 11 lb., tare being 14 lb. per cwt., and tret and cloff as usual?
3. What is the value of 15 packages of goods, each weighing 2 cwt. 1 qr. 4 lb., at 3*l.* 6*s.* 0*d.* per cwt.; tare being 2 cwt. 1 qr. 7 lb., and tret and cloff as usual?
4. What is the value of 5 casks of dried fruit, weighing 27 cwt. 1 qr. 11 lb., at five guineas per cwt., tare being 11 lb. on each cask, and tret and cloff as usual?
5. What is the cloff on 37 cwt., tare being 1 cwt. 1 qr. 9 lb., and tret as usual?
6. Find the value of 7 cwt. 2 qrs. 9 lb. at 7*l.* 10*s.* 6*d.* per cwt.?

BILLS OF PARCELS.

To make out a Bill, or Invoice, we have merely to work a number of sums in Practice, or Compound Multiplication, and then to add together the several results.

Example :

		s.	d.			s.	d.
5	pair of Stockings at	2	6	=		12	6
3	yards of Flannel ..	1	8	=		5	0
6	do. of Calico ..		6½	=		3	3
27	do. of Chintz ..	2	4	=	3	3	0
50	do. of Damask ..	3	6	=	8	15	0
					£12	18	9

Here the cost of each article, or set of articles, is first found, and noted down. And the total cost of the parcel is found by adding together the several sums so produced.

Exercise 113.

Make out the following Bills :

1. A Draper's Bill.

		s.	d.	
7	pair of Stockings, at....	2	4	=
5	yards of Satin, ...	9	6	=
12	do. of Glazed Calico, ...		6	=
18	do. of French Merino,.........	7	6	=
28	do. of Diaper, ...	1	9	=

2. A Grocer's Bill.

		s.	d.	
25	lb. of Fine Black Tea, at....	5	3	=
13	do. of do. Green do.	7	6	=
24	do. of Loaf Sugar,.............		9	=
12	do. of Cocoa,..................	1	4	=
20	do. of Raisins, ...		10½	=

3. A Wine Merchant's Bill.

	s.	d.	
12 doz. of Superior old Port, at ...	54	0	=
12 do. Sherry, at	66	0	=
13 gallons of Brandy, at..........	22	0	=
5 do. Rum, at	17	6	=
2 doz. of Claret, at	98	0	=
1 doz. of Fine Champagne, at	150	0	=

4. What is the amount of tret on 12 cwt. 1 qr. 9 lb., tare being 8 lb. per cwt.?

5. What is the cost of 5 cwt. 2 qrs. 19 lb., at 3*l*. 12*s*. 6*d*. per cwt., tare being 3 qrs. 5 lb.?

6. If 20 men can do a piece of work in 5 days, how many men will it require to do ten times the amount of work in 4 days?

END OF PART I.

ANSWERS TO THE EXERCISES.

Exercise 1.

1=29. 2=34. 3=43. 4=67. 5=84. 6=13.
7 = 70. 8 = 81. 9 = 76. 10 = 54. 11 = 68.
12 = 43. 13 = 122. 14 = 748. 15 = 516.

Exercise 2.

1 = 1174. 2 = 18,145. 3 = 27,912. 4 = 814,267.
5 = 9149. 6 = 147,142.

Exercise 3.

1 = 2,841,414. 2 = 8,967,213. 3 = 5,725,179.
4 = 68,242,674. 5 = 187,924,561. 6 = 346,597,863.

Exercise 4.

1 = four thousand. 2 = three hundred million.
3 = fifty thousand trillion. 4 = 34. 5 = 49. 6 = 63.
7 = 75. 8 = 123,216. 9 = 8421.

Exercise 5.

1 = twenty billions. 2 = six hundred thousand quadrillions. 3 = four thousand billions. 4 = 24,563.
5 = 187,428.

Exercise 6.

1 = eight thousand million. 2 = five hundred thousand.
3 = seven hundred million. 4 = sixty thousand million.
5 = nine thousand billions.

Exercise 7.

1 = 231,000. 2 = 340,000. 3 = 1,400,000.
4 = 33,000,000. 5 = 540,000,000. 6 = 4,310,000,000.

Exercise 8.

1 = 4,000,000,000,000. 2. = 57,000,000,000,000.
3 = 829,000,000,000,000. 4 = 9,400,000,000,000,000.
5 = 23,000,000,000,000,000,000.
6 = 333,000,000,000,000,000,000.

Exercise 9.

1=87,040. 2=190,024. 3=800,102. 4=4,000,076.

Exercise 10.

1 = 8,001,004. 2 = 120,034,000. 3 = 804,011,063.
4 = 900,000,900,042.

Exercise 11.

1 = 51. 2 = 53. 3 = 37. 4 = 175,000.
5 = 1,800,002.

Exercise 12.

1 = 13. 2 = 40. 3 = 27. 4 = 29. 5 = 46.
6 = 16,074. 7 = 2,024,000. 8 = 84,029,014.

Exercise 13.

1 = 97. 2 = 79. 3 = 55. 4 = 89. 5 = 201.
6 = 468. 7 = six hundred trillion. 8 = seven thousand
billions. 9 = ninety billions.

Exercise 14.

3 = 195. 4 = 246. 5 = 174. 6 = 163. 7 = 282.
8 = 361.

Exercise 15.

1 = 222. 2 = 144. 3 = 3032. 4 = 4472.
5 = 192.

Exercise 16.

1 = 3406. 2 = 2055. 3 = 395,144. 4 = 9,523,604.

Exercise 17.

1 = 9,862,048. 2 = 90,424,542. 3 = 169,930.
4 = 12,325. 5 = 1892.

Exercise 18.

1 = 18,593,517. 2 = 17,931,587. 3 = 727,19[illegible]
4 = 26,279,099. 5 = 164,507,672. 6 = 53,41[illegible]
7 = 45,600. 8 = 642,862.

Exercise 19.

1=11. 2=21. 3=81. 4=15,237. 5=211,964.
6 = 232,238. 7 = 305,002,027. 8 = 98,027,200.

Exercise 20.

1 = 11,011. 2 = 32,125. 3 = 1,122,770.

Exercise 21.

1 = 3181. 2 = 1154. 3 = 2011. 4 = 10,362.
5 = 16,098. 6 = 797,026. 7 = 27,440,066.
8 = 42,032,086.

Exercise 22.

1 = 2,864,701. 2 = 2,408,661. 3 = 199,054,378.
4 = 1,206,078,211,235. 5 = 2083. 6 = 2513, and 430.
7 = 738. 8 = 1921 : 4004 : 1012 : 8,000,032.
9 = 478,600*l.*

Exercise 23.

1 = 828. 2 = 948. 3 = 16,712. 4 = 308,820.
5 = 488,472. 6 = 537,128. 7 = 642,684. 9 = 51.

Exercise 24.

1 = 13,593,094. 2 = 11,893,977. 3 = 15,025,996.
4 = 438,327,195. 5 = 45,278,376. 6 = 59,258,465.
7 = 70,123,512. 8 = 716,894,685. 9 = 2040*l.*

Exercise 25.

1 = 4183. 2 = 714,210. 3 = 79,563,440.
4 = 71,473,215. 5 = 45,059,556. 6 = 44,258.
7 = 187,431. 8 = 3020. 9 = 5,064,000.
10 = 11,100,070. 12 = 0.

Exercise 26.

1 = 278,395. 2 = 8,676,336. 3 = 9,770,976.
4 = 2,633,988. 5 = 1,979,838. 6 = 1,724,259.

Exercise 27.

1 = 6,578,334. 2 = 88,035,600. 3 = 793,570,674.
4 = 54,759,803. 5 = 1. 6 = 101,011.

Exercise 28.

1 = 555,919,467,264. 2 = 197,346,076,210.
3 = 53,830,144,097,967. 4 = 1,009,447,638,232,152.

Exercise 29.

1 = 450,607,368. 2 = 5,458,714,830.
3 = 18,410,924,608. 4 = 30,755,190,690.
5 = 5500. 6 = 8,110,800.

Exercise 30.

1 = 8141. 2 = 37,171. 3 = 87,145.
4 = 25,613,634. 5 = 1,148,553. 6 = 545,269,824.

Exercise 31.

1 = 49,247,286,060. 2 = 6,575,523,815.
3 = 614,175. 4 = 6,141,627. 5 = 161,760,041.
6 = 436,575,037. 7 = 3,026,281,549.

Exercise 32.

1 = 687. 2 = 1753. 3 = 62,714. 4 = 81,214.
5 = 31,426. 6 = 271,416. 7 = 250. 8 = 138.

Exercise 33.

1=21,415. 2=361,514. 3=31,426. 4=814,127.
5 = 214,173,254. 6 = 81,214. 7 = 232,687.
8 = 201,047. 9 = B 2412*l.*; C 3618*l.*; capital, 7236*l.*

Exercise 34.

1 = 271,715. 2 = 8121. 3 = 214,167.
4 = 9,141,472. 5 = 814,126. 6 = 1,979,838.
7 = 8,004,639,884. 8 = 827*l.*

Exercise 35.

1=9,121,502. 2=2,717,150. 3=214,152,872,165.
4 = 20,702. 5 = 60,104. 6 = 1664*l.* 7 = 10,991,980.
8 = 49,099,960.

Exercise 36.

1 = 702,104. 2 = 61,402,015. 3 = 1,047,014,259.
4 = 62,714. 5 = 16,397,240. 6 = 200,239.

Exercise 37.

1 = 785,432 $\frac{16}{36}$. 2=472,849 $\frac{6}{45}$. 3=9248,374 $\frac{13}{108}$.
4 = 176,366,841 $\frac{3}{7}$.

Exercise 38.

1 = 61,428. 2 = 309,007. 3 = 90,806.
4 = 517,550. 5 = 3,425,928. 6 = 127,400,818.

Exercise 39.

1 = 13,520$\frac{12}{20}$. 2 = 32,347 $\frac{65}{300}$. 3 = 720.
4 = 815,036. 6 = 2,700,000. 7 = 87,915,994.

Exercise 40.

1 = 8,057,069. 2 = 926,785 $\frac{156}{400}$. 3 = 9.
4 = 120,000. 5 = 7,170,348,909. 6 = 59,024,109.
7 = 50,193,579.

Exercise 41.

1 = 71*l.* 15*s.* 1*d.* 2 = 197*l.* 3 = 1621*l.* 14*s.* 6*d.*
4=81,174,783,936. 5=1,808,030,240. 6=3,708,787.
7 = 89,817.

Exercise 42.

1 = 626*l.* 18*s.* 5$\frac{1}{2}$*d.* 2 = 1591*l.* 9*s.* 4$\frac{1}{4}$*d.*
3=834*l.* 8*s.* 4$\frac{3}{4}$*d.* 4=14,426*l.* 4*s.* 6$\frac{1}{2}$*d.* 5=4,654,537.
6 = 900,761,382,528.

Exercise 43.

1 = 790*l.* 18*s.* 9$\frac{1}{2}$*d.* 2 = 3827*l.* 8*s.* 7$\frac{1}{2}$*d.*
3 = 37,409*l.* 17*s.* 9$\frac{1}{2}$*d.* 4 = 52,186, rem. 193.
5 = 612,170, rem. 158.

Exercise 44.

1 = 160,491*l.* 0*s.* 5$\frac{3}{4}$*d.* 2 = 22,314,092*l.* 7*s.* 9*d.*
3 = 864,225*l.* 6*s.* 9$\frac{1}{2}$*d.* 4 = 28,291, rem. 1539.
5 = 31,906, rem. 730. 6 = 98,714,703, rem. 8.

Exercise 45.

1 = 39 mo. 3 w. 2ds. 2 = 734 yrs. 0 mo. 2w. 1 day.
3 = 29 mo. 3 w. 4d. 11 h. 4 = 430*l.* 13*s.* 9½*d.*
5 = 31 w. 6 d. 5 h. 1 min. 6 = 59,009*l.* 11*s.*

Exercise 46.

1 = 492 cwt. 2 qrs. 17 lb. 2 = 317 cwt. 1 qr. 3 lb. 3 oz.
3 = 7569 cwt. 0 qr. 20 lb. 10 oz.
4 = 58 d. 23 h. 11 min. 44 sec.
5 = 2454 yrs. 11 mo. 0 w. 6 d. 0 h. 41 min. 21 sec.
6 = 1,114,016, rem. 20. 7 = 8,040,380,865.
8 = 908,190,776.

Exercise 47.

1 = 436 lb. 9 oz. 4 dwt. 21 grs. 2 = 7673*l.* 8*s.* 8¾*d.*
3 = 605 lbs. 3 oz. 10 dwt. 15 grs.
4 = 160 d. 18 h. 39 min. 14 sec. 5 = 82 oz. 1 dwt. 18 gr.
6 = 1716 lb. 3 oz. 18 dwt. 7 = 28*l.* 18*s.* 7¼*d.*
8 = 1*l.* 10*s.* 6*d.*

Exercise 48.

1 = 104 lb. 9 ℥ 0 ʒ 2 ℈ 1 gr.
2 = 901 lb. 3 ℥ 2 ʒ 0 ℈ 4 gr. 3 = 97 lb. 6 ℥ 6 ʒ.
4 = 596 cwt. 0 qr. 3 lb. 1 oz. 5 = 6398*l.* 11*s.* 0¾*d.*
6 = 140 h. 15 min. 19 sec.

Exercise 49.

1 = 546 qr. 3 bu. 0 pk. 1 gal.
2 = 891 qr. 6 bu. 0 pk. 5 qts. 3 = 32,662*l.* 16*s.* 8½*d.*
4 = 778 lb. 8 ℥ 3 ʒ 2 ℈ 8 gr.
5 = 28,090 cwt. 3 qrs. 26 lb. 5 oz. 9 drs.
6 = 3242 qrs. 5 bu. 2 pk. 5 qts. 7 = 2517. 8 = 2266.

Exercise 50.

1 = 609 hhd. 3 gal. 2 qts. 1 pt.
2 = 598 pipes 1 hhd. 33 gal. 3 qts.
3 = 1528 tun. 2 hhd. 16 gal.
4 = 266 pun. 1 tierce, 9 gal. 3 qts.
5 = 322 lb. 0 ℥ 0 ʒ 2 ℈. 6 = 167 lb. 0 oz. 1 dwt. 19 gr.
7 = 8121. 8 = 214,167.

Exercise 51.

1 = 7 butts, 1 hhd. 31 gal. 1 qt.
2 = 385 hhd. 4 fir. 7 gal. 2 qts. 3 = 3042*l.* 19*s.* 0½*d.*
4 = 2441 ton, 19 cwt. 92 lb. 7 oz. 5 = 489 lb. 8 ℥ 3 ʒ.
6 = 446 lb. 9 oz. 9 dwt. 8 gr. 7 = 694. 8 = 261.
9 = 201,504. 10 = 880,356,600.

Exercise 52.

1 = 419 E. e. 2 qrs. 2 na. 1½ in.
2=516 Fl. e. 2 qr. 3 na. 3=638 yds. 2 qrs. 2 na. 1½ in.
4=4582*l.* 14*s.* 4¾*d.* 5 = 2436 tons 2 cwt. 0 qr. 10 lb.
6 = 868 yrs. 8 mo. 19 d. 13 h.
7 = 1409 yds. 0 qr. 1 na. 2 in.
8 = 528 ton, 15 cwt. 0 qr. 18 lb. 8 oz. 9 dr.

Exercise 53.

1 = 412 deg. 31 fur. 14 p. 1 yard.
2 = 3664 mil. 1 fur. 12 r. 9 ft. 3 = 2231 yds. 2 ft. 3 in.
4 = 7 deg. 9 mil. 5 fur. 0 p. 3 yds. 5 = 24,807,873,237.
6 = 16,624,290,474. 7 = 814,147. 8 = 270,406.

Exercise 54.

1 = 182 chal. 5 sa. 2 = 2562 chal. 6 sa. 1 bu.
3 = 3967 chal. 7 sa. 1 bu. 3 pk. 4 = 156*l.* 16*s.*
5 = 38*l.* 3*s.* 7½*d.* 8 = 218,814*l.* 11*s.* 6*d.*

Exercise 55.

1 = 1685 acres, 2 r. 33 p. 2 = 2067 acres, 2 r. 28 p.
3 = 3052 acres, 1 r. 29 p. 4 = 2485*l.* 13*s.* 5¾*d.*
5 = 37,741*l.* 15*s.* 5¾*d.* 6 = 1208 chal. 30 bu. 3 pk.
7 = 74,891. 8 = 60,054,012.

Exercise 56.

1=232*l.* 7*s.* 4*d.* 2=131*l.* 2*s.* 3½*d.* 3=411*l.* 6*s.* 3½*d.*
4 = 112*l.* 4*s.* 3*d.* 5 = 922*l.* 2*s.* 3¼*d.* 6 = 209*l.* 7*s.* 2*d.*
7 = 1422*l.* 1*s.* 6½*d.* 8 = 198 cwt. 1 qr. 1 lb.

Exercise 57.

1 = 194*l.* 16*s.* 11*d.* 2 = 757*l.* 13*s.* 0$\frac{3}{4}$*d.*
3 = 1328*l.* 10*s.* 7*d.* 4 = 8508*l.* 17*s.* 0$\frac{3}{4}$*d.*
5 = 29,157*l.* 8*s.* 10$\frac{3}{4}$*d.* 6 = 184 years 4 mo. 2 w. 2 da.
7 = 6 years 10 mo. 16 da. 8 = 11 years 5 mo. 7 da.
9 = 1830*l.* 15*s.* 4$\frac{3}{4}$*d.*

Exercise 58.

1 = 2 mo. 0 w. 5 d. 2 = 29 years 1 mo. 0 w. 1 da.
3 = 107*l.* 14*s.* 3$\frac{1}{4}$*d.* 4 = 6*l.* 2*s.* 5$\frac{3}{4}$*d.* 5 = 22*l.* 5*s.* 9*d.*
6 = 16,797*l.* 18*s.* 8*d.* 7 = 21*l.* 16*s.* 7$\frac{1}{2}$*d.*

Exercise 59.

1=132 cwt. 2 qr. 3 lb. 2=8 ton 7 cwt. 1 qr. 27 lb.
3 = 50 cwt. 2 qr. 23 lb. 11 oz. 4 = 116*l.* 3*s.* 8$\frac{1}{4}$*d.*
5 = 390*l.* 17*s.* 9*d.* 6 = 6 h. 17 min. 8 sec.
7 = 12,755*l.* 9*s.* 1$\frac{3}{4}$*d.* 8 = 41 min. 30 sec.

Exercise 60.

1 = 15 E. e. 3 qr. 3 na. 2 = 212 yds. 0 qr. 1 na.
3 = 1 lb. 9 ℥ 5 ʒ 1 ℈. 4 = 501*l.* 1*s.* 10$\frac{1}{4}$*d.*
5 = 675*l.* 11*s.* 8*d.* 6 = 3 hours 3 min.
7 = 42,870*l.* 16*s.* 6*d.* 8 = 22 miles 0 fur. 13 p.

Exercise 61.

1 = 143*l.* 5*s.* 2 = 21*l.* 18*s.* 3*d.* 3 = 1157*l.* 3*s.* 6*d.*
4 = 5796*l.* 2*s.* 8*d.* 5 = 111*l.* 7*s.* 6*d.* 6 = 192*l.* 15*s.* 6*d.*

Exercise 62.

1 = 59 mo. 3 w. 1 da. 2 = 17 yrs. 8 mo. 3 w. 1 da.
3 = 133 yrs. 6 mo. 4 = 603 yrs. 2 mo. 3 w.
5 = 29 yrs. 1 mo. 1 w. 6 = 142*l.* 18*s.* 8$\frac{1}{2}$*d.*
7 = 73*l.* 18*s.* 9$\frac{1}{2}$*d.* 8 = 33,521*l.* 16*s.* 3*d.* 9 = 7775*l.*

Exercise 63.

1 = 323 cwt. 3 qrs. 2 = 1019 cwt. 0 qrs. 16 lb.
3 = 14,797 cwt. 1 qr. 8 lb. 8 oz. 4 = 205 cwt. 3 qrs. 5 lb.
5 = 3*l.* 3*s.* 4$\frac{1}{2}$*d.* 6 = 37,847,572,849,755. 7 = 895.
8 = 311*l.* 4*s.* 8$\frac{1}{4}$*d.*

Exercise 64.

1 = 41*l.* 2*s.* 9*d.*   2 = 138 cwt. 1 qr. 26 lb.
3 = 139*l.* 3*s.* 5¼*d.*   4 = 3*l.* 15*s.*   5 = 17*l.* 2*s.* 3*d.*
6 = 12,663*l.* 10*s.*   7 = 5022*l.* 6*s.* 6*d.*
8 = 3338*l.* 6*s.* 10½*d.*

Exercise 65.

1 = 37*l.* 16*s.*   2 = 55*l.* 1*s.* 4*d.*   3 = 123*l.* 4*s.*
4 = 26,683*l.* 15*s.* 4*d.*   5 = 87,474*l.*   6 = 29,444*l.* 8*s.*
7 = 64*l.* 4*s.*   8 = 315*l.* 9*s.*

Exercise 66.

1 = 37*l.* 16*s.*   2 = 262 miles 4 fur.   3 = 8*l.* 17*s.* 4*d.*
4 = 30*l.* 0*s.* 0¼*d.*

Exercise 67.

1 = 55*l.* 3*s.* 1¼*d.*   2 = 1393*l.* 0*s.* 8*d.*   3 = 13*l.* 4*s.*
4 = 1082*l.* 11*s.* 9*d.*   5 = 42,870*l.* 16*s.* 6*d.*
6 = 178*l.* 0*s.* 10*d.*   7 = 6213*l.* 14*s.* 3*d.*

Exercise 68.

1 = 4089*l.* 7*s.*   2 = 204,361*l.* 3*s.* 11½*d.*
3 = 3,581,284*l.* 17*s.*   4 = 3,317,110*l.* 2*s.* 8*d.*
5 = 172,428*l.* 11*s.* 7½*d.*   6 = 2268*l.* 8*s.* 7¾*d.*
7 = 12 cwt. 2 qrs. 19 lb.   8 = 36 yrs. 10 mo. 20 d.

Exercise 69.

1 = 71*l.* 12*s.* 6*d.*   2 = 370*l.* 18*s.* 4¼*d.*   3 = 81*l.* 9*s.* 0¾*d.*
4 = 49*l.* 13*s.* 6*d.*   5 = 2354*l.* 3*s.* 3*d.*
6 = 2527*l.* 13*s.* 10*d.*   7 = 53,838*l.* 10*s.* 6*d.*

Exercise 70.

1 = 64 cwt. 3 qrs.   2 = 127 cwt. 1 qr. 16 lb.
3 = 120 yrs. 7 mo. 3 w.   4 = 128*l.* 11*s.* 6*d.*
5 = 724*l.* 10*s.* 4*d.*   6 = 17 ton 14 cwt. 1 qr. 3 lb.
7 = 2024*l.* 11*s.* 3*d.*   8 = 4*l.* 11*s.* 5*d.*

Exercise 71.

1 = 48*l.* 13*s.* 2*d.* 2 = 47*l.* 1*s.* 4½*d.*
3 = 54*l.* 15*s.* 10¼*d.* 4 = 6*l.* 12*s.* 5*d.* 5 = 19*s.* 0½*d.*
6 = 347*l.* 14*s.* 6*d.* 7 = 325*l.* 16*s.* 3*d.* 8 = 8 hrs. 8 min.
9 = (1) 56,128*l.* 3*s.* 4*d.*, and (2) 2760*l.* 12*s.* 8*d.*

Exercise 72.

1 = 8 cwt. 1 qr. 14 lb. 2 = 9 cwt. 2 qr. 7 lb.
3 = 9 mo. 3 w. 6 d. 4 = 12 lb. 9 ℥ 7 ʒ.
5 = 21*l.* 18*s.* 3*d.* 6 = 2*l.* 5*s.* 4¼*d.* 7 = 19*s.* 3¼*d.*
8 = 4624*l.* 0*s.* 3¼*d.*

Exercise 73.

1 = 3*l.* 11*s.* 1*d.* 2 = 229*l.* 7*s.* 8½*d.* 3 = 682*l.* 7*s.* 3½*d.*
4 = 5*l.* 15*s.* 10¼*d.* 5 = 19*s.* 9½*d.* 6 = 138*l.* 4*s.* 4¾*d.*
7 = 28,210*l.* 15*s.* 7½*d.* 8 = 299*l.* 6*s.* 0½*d.*

Exercise 74.

1 = 38 d. 7 h. 39 min. 53 sec. 2 = 2353 d. 21 h.
3 = 259 hhd. 28 gal. 1 qt. 1 pt. 4 = 4 yds. 0 qr. 3 na.
5 = 2311 miles 0 fur. 16 p. 6 = 201 lb. 9 ℥ 4 ʒ 0 ℈ 16 grs.
7 = 1 mo. 23 d. 8 = 6387 miles 4 fur.

Exercise 75.

1 = 8 cwt. 1 qr. 14 lb. 2 = 9 mo. 3 w. 6 d.
3 = 9 cwt. 2 qr. 7 lb. 4 = 12 lb. 9 ℥ 7 ʒ.
5 = 6031*l.* 10*s.* 6 = 157,680. 7 = 13*l.* 13*s.* 9*d.*
8 = 21,927*l.* 7*s.* 6*d.*

Exercise 76.

1 = 5*l.* 10*s.* 2¾*d.* 2 = 118*l.* 12*s.* 5*d.*
3 = 505*l.* 12*s.* 9¾*d.* 4 = 1123*l.* 14*s.* 10¼*d.*
5 = 28,210*l.* 15*s.* 7½*d.* 6 = 5*s.* 5*d.*
8 = 296 cwt. 1 qr. 21 lb.

Exercise 77.

1 = 1680. 2 = 200. 3 = 8640. 4 = 6960.
5 = 11,200. 6 = 2,369,760. 7 = 7,821,120.
8 = 103,680. 9 = 720.

Exercise 78.

1 = 516. 2 = 7056. 3 = 468. 4 = 2555. 5 = 864.
6 = 83,520. 7 = 302,400. 8 = 192.

Exercise 79.

1 = 30,516. 2 = 3938. 3 = 600. 4 = 675.
5 = 1095. 6 = 25,920. 7 = 474. 8 = 0.

Exercise 80.

1 = 2*l.* 2 = 3*l.* 3 = 29*l.* 4 = 9874*l.* 5 = 4*l.*
6 = 44,042. 7 = 95,999. 8 = 3.

Exercise 81.

1 = 1254. 2 = 435. 3 = 2003. 4 = 21.
5 = 630 shillings 7560 pence. 6 = 0. 7 = 79.
8 = 159,022.

Exercise 82.

1 = 23*l.* 14*s.* 2 = 99*l.* 19*s.* 11*d.* 3 = 27 lb. 7 ℥ 2 ʒ 1 ℈.
4 = 16. 5 = 36. 6 = 330. 7 = 320.
8 = 4330*l.* 0*s.* 10*d.* 9 = 6*l.* 4*s.* 9½*d.*

Exercise 83.

1 = 8. 2 = 27. 3 = 24. 4 = 97. 5 = 24.
6 = 27. 7 = 1*l.* 11*s.* 6*d.* 8 = 9*s.* 5½*d.* 9 = 437*l.* 8*s.*

Exercise 84.

1 = 21. 2 = 30 bu. 2 gal. 4 pts. 3 = 8766.
4 = 7602. 5 = 8746. 6 = 1198. 7 = 240.
8 = 5173*l.* 6*s.* 8*d.* 9 = 1116*l.* 6*s.* 10*d.*

Exercise 85.

1 = 21 ton 15 cwt. 2 = 17 cwt. 3 qrs. 15 lb.
3 = 4 yds. 3 qrs. 3 na. 4 = 26 yds. 3 qrs.
5 = 26 Fl. e. 2 qrs. 6 = 16 miles 0 fur. 8 p.
7 = 136*l.* 17*s.* 6*d.*
8 = 683,280 (at 6 working days to a week).

Exercise 86.

1 = 16. 2 = 927. 3 = the same. 4 = 3550.
5 = 3550 miles 2 fur. 32 p. 6 = 927. 7 = 22,377.
8 = 4,755,801,600.

Exercise 87.

1 = 90,544,264. 2 = 8820. 3 = 70.
4 = 148 pipes 1 hhd. 29 gal. 3 qts. 5 = 36. 6 = 8746.
7 = 1. 8 = 224 hhd. 40 gal. 1 qt. 9 = 262,691.

Exercise 88.

1 = 2*l.* 16*s.* 2 = 30. 3 = 15. 4 = 108.
5 = 34,992. 6 = 2800.

Exercise 89.

1 = 6*l.* 12*s.* 2 = 7*s.* 3 = 220*l.* 4 = 1173*l.*
5 = 500. 6 = 1020*l.* 7 = 32,000.
8 = 1 lb. 10 ℥ 3 ʒ 0 ℈ 1 gr. 9 = 37 lb. 7 ℥ 1 ʒ 1 ℈.

Exercise 90.

1 = 8*l.* 6*s.* 6*d.* 2 = 11*l.* 13*s.* 9*d.* 3 = 1*l.* 11*s.* 6*d.*
4 = 3*l.* 2*s.* 5 = 48. 6 = 26 yrs. 3 mo.
7 = 5 lb. 2 oz. 14 dwts. 8 = 520.

Exercise 91.

1 = 326*l.* 5*s.* 2 = 1458*l.* 3 = 19 lb. 5 oz. 5 drs.
4 = 800. 5 = 55 fur. and 1540 miles.
6 = 93*l.* 15*s.* 7 = 87,342*l.* 13*s.* 4*d.*

Exercise 92.

1 = 3*s.* 9½*d.* 2 = 1¼*d.* 3 = 98*l.* 15*s.* 4 = 3*s.*
5 = 99*l.* 16*s.* 10*d.* 6 = 1536. 7 = 111*l.* 12*s.* 2½*d.*
8 = 8918.

Exercise 93.

1 = 42750. 2 = 55*l.* 3*s.* 7½*d.* 3 = 2*l.* 10*s.* 0½*d.*
4 = 2797*l.* 10*s.* 6¾*d.* 5 = 101 miles 6 fur. 6 = 2880.
7 = 9,426,848. 8 = 21.

Exercise 94.

1=289*l.* 2*s.* 2=10*l.* 2*s.* 10*d.* 3=1133*l.* 19*s.* 6¾*d.*
4 = 12 w. 3 d. 5 h. 32 min. 5 = 1993*l.* 16*s.* 3*d.*
6 = 146 yds. 2 ft. 9 in. 1 b. 7 = 57,105*l.* 10*s.* 8*d.*
8 = 127*l.* 15*s.* 8*d.*

Exercise 95.

1=1200*l.* 2=96. 3=7*l.* 12*s.* 7¾*d.* 4=1975.
5 = 179*l.* 7*s.* 1½*d.* 6 = 6750*l.* 7 = 46*l.* 11*s.* 10½*d.*
8 = 267 cwt. 1 qr. 11 lb.

Exercise 96.

1 = 66 bu. 1 pk. 2 = 43 bar. 24 gal. 3 = 600*l.*
4 = 77 d. 7 h. 12 m. (at twelve hours to the day.)
5 = 533 lb. 5 oz. 5 dr. 6 = 6. 7 = 927. 8 = 8640.

Exercise 97.

1 = 191 days 5 hrs. (nearly.) 2 = 300. (nearly.)
3 = 24 days 4 hrs. 4 = 12 days 12 hrs. 5 = 672*l.*
6 = 8*s.* 3*d.* 7 = 76*l.* 13*s.* 4*d.* 8 = 3047*l.* 8*s.* 9*d.*

Exercise 98.

1 = 3*l.* 4*s.* 2½*d.* 2=18*l.* 8*s.* 7½*d.* 3=2723*l.* 3*s.* 3*d.*
4=102*l.* 4*s.* 7¼*d.* 5=27,231*l.* 15*s.* 3¾*d.* 6=1680*l.*
7 = 352*l.* 16*s.* 8*d.* 8 = 10*l.* 11*s.* 3*d.*

Exercise 99.

1 = 5562*l.* 5*s.* 2*d.* 2 = 1999*l.* 14*s.* 6*d.*
3 = 1341*l.* 19*s.* 3*d.* 4 = 3,149,398*l.* 11*s.* 4*d.*
5 = 6,915,866*l.* 0*s.* 3½*d.* 6 = 41,035,505*l.* 8*s.* 1*d.*
7 = 49*l.* 14*s.* 6*d.* 8 = 4½*d.* 9 = 7311*l.* 8*s.* 5½*d.*

Exercise 100.

1 = 2968*l.* 5*s.* 1¼*d.* 2 = 1842*l.* 13*s.* 6*d.*
3 = 2,903,528*l.* 2*s.* 10½*d.* 4 = 9596*l.* 1*s.* 6¾*d.*
5 = 3,343,621*l.* 7*s.* 4½*d.* 6 = 398,895,576*l.* 2*s.* 7¾*d.*
7 = 6*s.* 3*d.* 8 = 10*d.* 9 = 74 lb. 9 oz. 10 dwt. 9 grs.

Exercise 101.

1 = 737*l.* 13*s.* 9*d.* 2 = 11,780*l.* 12*s.* 6*d.*
3 = 338,389*l.* 3*s.* 4*d.* 4 = 3,032,353*l.* 15*s.* 6*d.*
5 = 1,967,777*l.* 12*s.* 6 = 7,532,195*l.* 6*s.* 7*d.*
7 = 15*l.* 17*s.* 4*d.* 8 = 18*l.* 18*s.* 9*d.* 9 = 37*l.* 16*s.*

Exercise 102.

1 = 6648*l.* 9*s.* 8*d.* 2 = 75,754*l.* 15*s.* 2½*d.*
3 = 214,870*l.* 4*s.* 2*d.* 4 = 1821,693*l.* 3*s.*
5 = 8836*l.* 5*s.* 1½*d.* 6 = 2856*l.* 15*s.*
7 = 65,400. 8 = 264. 9 = 5*l.* 9*s.* 6¾*d.*

Exercise 103.

1 = 20,707,329*l.* 8*s.* 4*d.* 2 = 5,384,555*l.* 4*s.* 6*d.*
3 = 1,503,569*l.* 17*s.* 1*d.* 4 = 4,438,491*l.* 10*s.* 11½*d.*
5 = 81,794*l.* 5*s.* 11*d.* 6 = 2,942,789*l.* 1*s.* 11½*d.*
7 = 24*l.* 12*s.* 6*d.*
8 = 409 miles 0 fur. 20 p. (12 hours to the day.)

Exercise 104.

1 = 10,373,959*l.* 4*s.* 2 = 848,060*l.* 12*s.* 1*d.*
3 = 534,636*l.* 19*s.* 1¾*d.* 4 = 3,729,973*l.* 7*s.* 0¾*d.*
5 = 217,468*l.* 4*s.* 8¼*d.* 6 = 45 yds. 7 = 533⅓.
8 = 6*l.* 2*s.* 7¾*d.*

Exercise 105.

1 = 22*l.* 1*s.* 10½*d.* 2 = 3*l.* 8*s.* 2¼*d.*
3 = 373*l.* 10*s.* 10½*d.* 4 = 430*l.* 9*s.* 5¼*d.*
5 = 70*l.* 10*s.* 6*d.* 6 = 27*l.* 17*s.* 8¼*d.*
7 = 2 cwt. 1 qr. 17 lb. 6 oz. 13 drs.
8 = A = 250*l.* B = 166*l.* 13*s.* 4*d.* C = 83*l.* 6*s.* 8*d.*

Exercise 106.

1 = 121*l.* 4*s.* 3¾*d.* 2 = 910*l.* 19*s.* 1½*d.*
3 = 18*l.* 0*s.* 9¾*d.* 4 = 37*l.* 19*s.* 10½*d.*
5 = 123*l.* 6*s.* 6 = 6⅔. 7 = 2 lb. 11 oz. 8 = 2400.

Exercise 107.

1 = 17 cwt. 1 qr. 22 lb. 2 = 142 cwt. 2 qr. 19 lb.
3 = 145 cwt. 0 qr. 10 lb. 4 = 15 cwt. 2 qrs. 2 lb.
5 = 8 months. 6 = 489*l.* 11*s.* 8*d.*

Exercise 108.

1 = 17 cwt. 0 qrs. 4 lb. 2 = 43 cwt. 3 qrs. 12 lb.
3 = 199 cwt. 1 qr. 4 lb. 4 = 21 cwt. 2 qrs. 20 lb.
5 = 455*l.* 6 = 59*l.* 4*s.* 0*d.*

Exercise 109.

1 = 16 cwt. 1 qr. $8\frac{1}{2}$ lb. 2 = 12 cwt. 1 qr. 27 lb.
3 = 21 cwt. 2 qrs. 17 lb. 4 = 22 cwt. 2 qrs. 19 lb.
5 = 283*l.* 11*s.* $11\frac{1}{2}$*d.* 6 = 313*l.* 5*s.* 0*d.*

Exercise 110.

1 = 13 cwt. 0 qrs. $2\frac{1}{4}$ lb. 2 = 154 cwt. 2 qrs. $19\frac{1}{2}$ lb.
3 = 224*l.* 8*s.* 9*d.* 4 = 241*l.* 7*s.* $10\frac{3}{4}$*d.*
5 = 2*l.* 19*s.* 6*d.* 6 = 6*l.* 19*s.* $4\frac{1}{2}$*d.*

Exercise 111.

1 = 7 cwt. 0 qrs. $26\frac{3}{4}$ lb. 2 = 4 cwt. 2 qrs. 20 lb.
3 = 55 cwt. 2 qrs. $24\frac{1}{2}$ lb. 4 = 1 cwt. 0 qrs. 8 lb.
5 = 4*l.* 13*s.* $9\frac{3}{4}$*d.* 6 = 19 cwt. 2 qrs. $2\frac{1}{2}$ lb.

Exercise 112.

1 = 74 cwt. 3 qrs. $15\frac{1}{2}$ lb. 2 = 214 cwt. 1 qr. $17\frac{1}{4}$ lb.
3 = 100*l.* 17*s.* 5*d.* 4 = 134*l.* 15*s.* $9\frac{1}{4}$*d.*
5 = $22\frac{3}{4}$ lb. 6 = 57*l.* 0*s.* $10\frac{1}{2}$*d.*

Exercise 113.

1 = 11*l.* 13*s.* 10*d.* 2 = 14*l.* 0*s.* 3*d.* 3 = 107*l.* 19*s.* 6*d.*
4 = 1 qr. 21 lb. 5 oz. 5 = 18*l.* 11*s.* $6\frac{3}{4}$*d.* 6 = 250.

THE END.

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